Seismic design of reinforced soil structures using a two-part wedge method with special reference to the inclusion of vertical acceleration

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ABSTRACT: Vertical acceleration is likely to be major ground motion component in the epicentral areas of strong earthquakes, so should be taken into account in the design of reinforced soil structures. The magnitude of vertical acceleration which might be present at the same point in time as peak horizontal acceleration cannot be predicted but may well be significant. Vertical and horizontal acceleration may be included simultaneously in the internal stability analysis of reinforced soil structures using a two-part wedge method. This technique gives a good prediction of the behaviour of a full-scale reinforced soil retaining wall subjected to a maximum purely horizontal acceleration of 0.55g in a shaking table test. This same case is analysed with 0.55g applied as a vector with both horizontal and vertical components, in order to find the critical vector direction. This approach provides a basis for rational design recommendations under seismic conditions which take into account the vertical component of acceleration.

1 INTRODUCTION

Design methods for reinforced soil structures have been in use for almost 40 years, and in many cases design for the seismic loading case is based on US practice, which in the past has specifically omitted vertical acceleration. Advice in Section 11.6.5 of the latest version of the AASHTO LRFD Bridge Design Specifications (AASHTO, 2017) states that "the seismic vertical acceleration coefficient k_v should be assumed to be zero for the purpose of calculating lateral earth pressures, unless the wall is significantly affected by near fault effects, or if relatively high vertical accelerations are likely to be acting concurrently with the horizontal acceleration". The commentary on Section 11.6.5 further states that "in most situations, vertical and horizontal acceleration are at least partially out of phase, therefore k_v is usually rather small when k_h is near its maximum value and the typical assumption is to assume that k_v is zero for wall design". The consequence of this advice is that vertical acceleration is often ignored in the design of reinforced soil structures, despite the warning that it might be significant.

One of the conclusions from a major study of the effects of the destructive El-Asnam earthquake of 1980 (EERI, 1983), in which vertical acceleration was dominant in the epicentral area, was that comprehensive recommendations for seismic-resistant design of civil engineering structures should be developed based on local seismicity, which in the El-Asnam case should take account of the high vertical acceleration. More recently, observations from the 2010/11 Christchurch earthquakes and the 2016 Kaikoura earthquake in New Zealand indicate that peak vertical ground accelerations could be significantly greater than peak horizontal accelerations, especially at locations close to the epicentres, where shaking was most severe.

This paper examines the occurrence of vertical ground acceleration in a number of earthquakes in relation to the horizontal acceleration, in particular the component of vertical acceleration which occurs at the same instant as the peak horizontal acceleration. Following this, a method of calculation for the internal stability of reinforced soil structures based on a two-part wedge method is described, which can take account of both horizontal and vertical acceleration for the seismic design case. The method is used to back-analyse the behaviour of a 6m high reinforced soil retaining wall subjected to a maximum purely horizontal acceleration of 0.55g on a shaking table, giving very good agreement to the behaviour observed. This same case is then analysed with the acceleration applied as a vector with both horizontal and vertical components, in order to find the critical vector direction. Based on this analysis, advice is provided in terms of an appropriate approach to the design of the internal layout of reinforced soil structures in order to take account of the occurrence of vertical acceleration in seismic design.

2 RECORDS OF VERTICAL ACCELERATION IN EARTHQUAKES

2.1 Peak ground accelerations

Strong motion data, consisting of records of horizontal and vertical acceleration versus time, are available for many recent strong earthquakes. The Chi-Chi earthquake occurred on 21^{st} September 1999 resulting from rupture along the Chelungpu fault in central Taiwan. The moment magnitude of the main shock was $M_w = 7.7$, with a focal depth of 33 km. The motions generated by the main shock of this earthquake were recorded at 387 strong motion stations located throughout Taiwan. Idriss and Abrahamson (2000) give a summary of the strong motion data recorded, and Figure 1 shows the relationship of peak vertical ground acceleration to peak horizontal ground acceleration measured at several strong motion monitoring stations. Almost everywhere the horizontal acceleration is greater than the vertical component, but the vertical component is significant, with the mean ratio being greater than 0.5.

During 2010 and 2011, the city of Christchurch in the South Island of New Zealand was shaken by a series of earthquakes and a huge number of aftershocks. The second of the main earthquakes occurred on 21^{st} February 2011, with $M_w = 6.3$ and focal depth = 5 km. Although significantly smaller than the first shock, this earthquake caused major damage to the city centre of Christchurch, resulting in an extensive rebuilding programme. Like Taiwan, New Zealand has a network of strong motion recording stations, and data from the second main earthquake is shown in Figure 2. In this case the data is plotted as an attenuation relationship with the three measured peak components of strong motion (denoted as *H1* and *H2* for the two horizontal components and V for the vertical component) plotted against distance



Figure 1. Relationship between peak vertical and horizontal acceleration for Chi-Chi earthquake (1999).



Figure 2. Attenuation of peak vertical and horizontal acceleration for Christchurch earthquake (2011).

to the epicentre. At large distances from the epicentre, the horizontal components of acceleration are consistently higher than the vertical. However, within about 20 km this trend changes, and close to the epicentre, the vertical acceleration tends to dominate.

Figure 3 shows similar data for the Kaikoura earthquake which occurred on 14th November 2016 due to ruptures on faults along the east coast of the Upper South Island of New Zealand with $M_w = 7.8$ and depth = 15 km. It should be noted that the distance measurement is to the nearest point on the fault break, due to the mechanism of the fault rupture which took place (for a full explanation see Stevens & Dobie, 2019). The pattern of behaviour with regards to the vertical component of acceleration is similar to the Christchurch earthquake.

From these earthquake records it can be seen clearly that the peak vertical component of acceleration can be equal to or greater than the peak horizontal component, especially close



Figure 3. Attenuation of peak vertical and horizontal acceleration for Kaikoura earthquake (2016).

to the earthquake epicentre. Reinforced soil structures can withstand strong ground shaking very effectively, so from a design point-of-view, the earthquake loading case is only likely to be critical when the ground acceleration is high, so this will be close to the epicentre. This already indicates that the vertical acceleration component should be given careful consideration, for example in the bearing capacity calculation in the case where the foundation has a relatively low strength.

However, what is of major importance to design in general, and especially to the internal stability of a reinforced soil structure, is the combination of horizontal and vertical acceleration which should be included in design, as indicated in the advice in the AASHTO LRFD Bridge Design Specifications (AASHTO, 2017) mentioned in the introduction. This is investigated in the following section.

2.2 Vertical acceleration at the same moment as peak horizontal acceleration

New Zealand has an extensive system of stations which record the strong motion data from earth-quakes. Information from these recordings is freely available from the New Zealand Geonet website (ftp.geonet.org.nz). The data in Figures 2 and 3 are based on information from this source. In addition to information on peak ground acceleration, strong motion data for each recording station may be downloaded, both in graphical and digital form. Figure 4 shows part of the strong motion record measured during the Kaikoura earthquake at Seddon Fire Station, referred to as SEDS. This record was of particular interest being very close to a reinforced soil retaining wall which survived the earthquake with no sign of any damage, as described by Stevens & Dobie (2019).

Figure 4 only shows 10 seconds of data, during which time the peaks occur for all three measured accelerations. The measured acceleration values used to create the processed strong motion records are given at intervals of 0.02 secs, so that each one second of data consists of 50 data points, resulting in well-defined peaks and troughs in the records. In this case there are typically about 8 peaks per second. It is clear from Figure 4 that the peaks of the three components of strong motion occur at different points in time, although all within about a 0.7 second interval. However, whenever each peak occurs, there will be some component from each of the other acceleration records, although the magnitude will generally be less than the peak.

In order to get a more general impression from this data of the coincidence of the three peaks in such records with the values of the other components, 11 strong motion records have been examined in detail, five from the Kaikoura earthquake and six from the second Christchurch earthquake, and the data is summarised in Table 1. The chosen records are all close to the epicentres or fault-breaks of the two events, representing some of the strongest ground



Figure 4. Strong motion data from the Kaikoura earthquake recorded at Seddon Fire Station (SEDS).

		H1 _{max}	V1	H2 _{max}	V2	V _{max}
Earthquake	Station	m/sec ²				
Kaikoura	SEDS (Figure 4)	-5.98	-0.36	7.28	0.50	1.75
	MOLS	-3.13	1.15	2.69	1.15	1.51
	WDFS	11.91	0.27	-7.84	-1.08	3.49
	WTMC	-9.73	-4.14	7.97	-2.09	18.02
	KEKS	-11.68	2.11	-7.41	0.43	3.45
Christchurch	CCCC	-4.74	1.30	-3.60	-0.61	6.78
	CBGS	5.19	-1.46	-4.22	0.48	-2.65
	CHHC	-3.29	0.38	3.54	0.64	5.01
	CHMS	3.48	5.14	3.89	2.98	7.80
	PRPC	-6.52	1.53	-5.78	-0.48	15.98
	REHS	7.00	-0.81	-3.59	1.49	5.18

Table 1. Comparing peak horizontal acceleration to the vertical component at the same time.

motions measured, therefore sufficiently high to be critical in the design of reinforced soil structures. Interestingly, of the 11 records examined, one of them (MOLS, Kaikoura) recorded all three peaks within 0.02 seconds of each other. Table 1 lists the recording station (which includes Seddon Fire Station, SEDS), and then each value of peak ground acceleration ($H1_{max}$, $H2_{max}$ and V_{max}). Adjacent to the values of $H1_{max}$ and $H2_{max}$ are given the values of vertical acceleration occurring at the same point in time, listed as V1 and V2.

Examination of this data indicates very variable results, with no apparent pattern of behaviour. In order to see the relationships in a graphical form, the vertical acceleration values which coincide with each peak horizontal acceleration have been plotted against the peak horizontal acceleration in Figure 5. The inclined lines represent the relation between the two values, varying from $V = 0.1H_{max}$ to $V = 2H_{max}$. The scatter is very big, but importantly a substantial amount to the data plots around the $V = 0.3H_{max}$ line, suggesting that it can be anticipated that a significant vertical acceleration component may well occur at the same time as peak horizontal acceleration. Based on this, it can be concluded that combinations of vertical and horizontal acceleration should be investigated as part of the seismic design process for any reinforced soil structure.



Figure 5. Vertical acceleration at the same point in time as the peak horizonal accelerations.

3 DESIGN OF REINFORCED SOIL STRUCTURES FOR EARTHQUAKE LOADING

3.1 General approach and external stability

The general approach to the design of reinforced soil structures published in most codes and guidelines is divided into two main parts: external stability to determine the overall dimensions of the reinforced soil block and internal stability. For the external stability calculations, the block of reinforced soil is considered to act as a gravity retaining wall, which must be adequate for stability checks such as sliding on the base, bearing capacity and eccentricity of the resultant force on the base. The approaches to these calculations are much the same in all published guidelines, the differences being in the factors of safety or partial factors used to achieve a safe design, as well as other controlling parameters, such as wall friction angle. Some codes specify limitations on the length of the geogrid (L) in relation to the height of the wall (H), with the ratio L/H typically limited to 0.7. This requirement will often determine the overall dimensions of the reinforced soil block rather than the stability calculations.

In the case of designing for earthquake loading, the normal approach used is to add the additional loading created by the earthquake as static loads, so that the technique is called pseudo-static design. There are two components of additional load: inertia of the reinforced soil block itself and additional earth pressure on the back of the reinforced soil block. This approach is outlined in detail in Section 11.10.7.1 and Figure 11.10.7.1-1 of the AASHTO LRFD Bridge Design Specifications (AASHTO, 2017) and is beyond the scope of this paper. However, the approach is well suited to including vertical acceleration, which can be applied in the calculation of both the inertia and the earthquake induced additional earth pressure.

3.2 Internal stability using tie-back wedge

The purpose of the internal stability calculations is to determine the required layout of reinforcement (grade and spacing) to ensure that the stability of the block of reinforced soil itself is adequate. One method used in several design guides is the tie-back wedge method, in which all internal stability checks are carried out based on a single failure mechanism, normally the Rankine wedge, as shown in Figure 6. This is described in Section 11.10.6 of the AASHTO LRFD Bridge Design Specifications (AASHTO, 2017) for the static design case, while Section 11.10.7.2 and Figure 11.10.7.2-1 provide guidance on adapting this approach for the seismic design case. This adaptation includes several simplifying assumptions. Firstly, the size of the additional force due to the earthquake is calculated from the forward inertia (P_i) of the mass of the Rankine wedge as shown in Figure 6. Secondly, the distribution of that force between the layers of reinforcement is assumed to be uniform in the 2017 edition (in previous versions of the AASHTO guide distribution was assumed to be proportional to the length of each layer behind the active wedge, L_a in Figure 6). Thirdly a reduction is applied to the design acceleration used to calculate P_i due to wave scattering and lateral displacement, which is also a new assumption in the 2017 guide.



Figure 6. The basis of the tie-back wedge method for internal stability calculations.

These assumptions appear to be arbitrary and are not well substantiated. The Rankine wedge is only the critical failure wedge in the case of a homogeneous soil mass, without reinforcement, for the static loading case. Extending this to the case where reinforcement is present under seismic loading cannot be justified. Likewise, assuming a uniform distribution of the inertia force between the layers of reinforcement does not take account of varying stiffness, load-carrying capacity nor variations in spacing of the reinforcement. The third assumption is possibly the most concerning, permitting a reduction in acceleration on the basis that some deformation is acceptable during the earthquake event. This can be justified for external mechanisms, such as sliding on the base of the structure, in which the reinforcement is not intersected. However, looking at Figure 6, this assumption would imply some relative movement along the inclined plane forming the Rankine wedge, and should this happen, it would almost certainly result in rupture of the geogrid taking place, especially at greater burial depths. This assumption would therefore appear to be fail-unsafe.

With regards to vertical acceleration, the adapted tie-back wedge method offers few opportunities in the method of calculation for adding the effects of the additional vertical forces to the internal stability calculation. Vertical forces may be taken into account in assessing the pull-out resistance of the reinforcement behind the critical wedge and in checks of sliding over reinforcement, but not in any other aspects of the internal stability calculations.

All the issues outlined in the previous two paragraphs may be addressed satisfactorily by using a two-part wedge method of calculation, in which assumptions are minimised, and large numbers of possible failure mechanisms are examined in order to find the critical case. This technique also permits the inclusion of vertical acceleration in the seismic design case and is described in the following section.

3.3 Internal stability using two-part wedge

The two-part wedge method of internal stability analysis described in this section was first published in a DIBt Certificate (Deutsches Institut für Bautechnik, 1995), which provided the basis for the method of calculation. Since then, various adaptations have been made, adding the forces for the earthquake design case, as well as including a realistic approach to model the connection strength with the facing. Dobie (2011) provides a full description of the technique and Dobie (2014) provides a detailed description of the approach to the seismic design case, including back-analysis of three modular block retaining walls which were affected by the Chi Chi earthquake in 1999. The method of calculation may readily be adapted to various design codes, incorporating their definitions of partial factors and other requirements. Dobie (2012) describes the two-part wedge method combined with the advice given in the Australian retaining wall design code AS 4678-2002 (Standards Australia, 2002) and Dobie (2015) provides a similar outline for using the method according the requirements of AASHTO/LRFD, both being limit state design methods. Extensive justification for the technique, based both on analytical methods and observed behaviour is described by Dobie & McCombie (2015). To date, this design approach based on a two-part wedge technique has been used extensively to design a very large number of retaining walls in many different regions and countries, including some of the largest ever built in the Middle East.

This section provides a brief overview of the two-part wedge technique, and full information should be obtained from the references cited in the previous paragraph. One of the important targets of the method is to reduce the need for assumptions and simplifications to a minimum. Each mechanism analysed is complete with all disturbing forces and resistances taken into account, such that the method is one of limiting equilibrium, rather than an allowable stress design (ASD) approach. This is much the same as slope stability analysis, and in common with stability analysis techniques, no assumption is made about the critical mechanism before the analysis is made. Instead large numbers of possible failure mechanisms are investigated in order to establish the critical case or cases. The critical mechanism for the seismic loading case will be different to the static loading case, especially for strong ground motions.

The basis of the two-part wedge method of analysis for internal stability is shown on Figure 7 (left). The geometry is typical of reinforced soil structures, but the method of analysis



Figure 7. The basis of the two-part wedge method for internal stability calculation.

can incorporate all features shown (ie. berm, slope above the wall, isolated surcharge) without the need for any simplifying assumptions.

The two-part wedge is defined as follows with reference to Figure 7 (left): fix a distance H_i below the top of the wall, then draw a line at an angle θ_i across the reinforced soil block (RSB), defining Wedge 2. Starting at the point where Wedge 2 intersects the back of the reinforced soil block, define a second wedge, Wedge 1 as shown, with the inter-wedge boundary defined as the back of the reinforced soil block.

Wedge 1 is used to calculate the earth pressure forces applied to the back of the RSB, and for simple geometry and conditions, this may be replaced by the Coulomb formula (or Mononobe Okabe for the seismic design case). However, for the geometry and isolated surcharge as shown in Figure 7, it is not possible to use the Coulomb formula without making simplifying assumptions. In this situation, to obtain the maximum lateral forces applied by Wedge 1 rigorously, it is necessary to use a trial wedge method in which the angle of Wedge 1 is varied until the maximum lateral thrust is obtained (Culmann wedge method) as mentioned in AASHTO/LRFD Section 3.11.5.3. The aim of the calculation is to make sure that the resistance provided by the facing and reinforcement which is intersected by Wedge 2 ($T_1 + T_2 + T_3$ as shown on Figure 7) is sufficient to ensure stability of the two wedges.

In order to find the critical two-part wedge, it is necessary to search through many combinations of wedges. This is normally done as shown on Figure 7 (right). For a specific value of H_i various values of θ_i are used so that a "fan" of wedges is checked. H_i is then adjusted and the fan of wedges repeated. Normally H_i is chosen starting at the base of the wall (where $H_i = H$, the total wall height), then at each elevation where reinforcement intersects the facing.

The forces taken into account in the internal stability calculations are illustrated in Figure 8, with definitions and explanations provided in Table 2. Figure 8 (left) shows the forces for the static load case, whereas Figure 8 (right) includes the additional forces used to model the earthquake loading as pseudo-static loads. These additional forces have two components, consisting of the intertia of the mass of the front part of Wedge 2 and the dynamic increment of earth pressure on the back of Wedge 2. This dynamic increment is the difference between the full earth pressure including the earthquake induced forces, and the static earth pressure. Importantly, in the case where vertical acceleration is to be included in the analysis, both components of additional force may be modified for the effects of vertical acceleration. Vertical acceleration may be either up or down, and it is not immediately obvious which will be critical. Therefore, the calculations are carried out for both cases in order to find out which is critical. For internal stability analysis using this two-part wedge approach, steep wedges will generally be critical when k_v is down and low angle wedges when k_v up will normally be the critical case overall. However, it is important that for a complete analysis, k_v both up and down should be investigated.



Figure 8. The forces applied to Wedge 2 for the static case (left) and seismic case (right).

Forces for the static case		Additional forces for the seismic case (denoted by *)		
E _{ah}	Horizontal earth pressure force applied by retained backfill and any superimposed surcharges behind the reinforced soil block	E* _{ah}	Additional horizontal earth pressure force applied by retained backfill and any superimposed surcharges due to earthquake (dynamic increment)	
E_{av} W_i	Vertical component of E_{ah} Weight of Wedge 2	$E^*_{av} \\ k_h W^*_i$	Vertical component of E^*_{ah} Horizontal inertia of Wedge 2* defined by a width of 0.5 <i>H</i> from front of the facing	
		$k_v W^*_i$	Vertical inertia of Wedge 2* which can act either up or down. It is not certain which will be critical, so it is normal to check both and use the critical case	
<i>Q</i> ₂	Any surcharge applied to the top of the reinforced soil block. If Q_2 is a live load, then it is not immediately obvi- ous whether it should be included or not, so it is normal to check both with and without live load and use the crit- ical case	Q*2	Surcharges applied to the top of the reinforced soil block have both hori- zontal and vertical inertia	
R_i	Resistance on the base of Wedge 2	$R^*{}_i$	Resistance on the base of Wedge 2	

The method of analysis described above involves a huge number of calculations for a typical reinforced soil retaining wall, so it is not amenable to hand-calculation. Individual cases may be verified by hand-calculation and simple cases may be set up in a spreadsheet, but the technique is well suited to creating dedicated computer programs. These may have automatic search routines to find the optimum layout of reinforcement, making the process extremely quick and, thereby encouraging the use of sensitivity analysis if required.

It should also be noted that all analyses of mechanisms which intersect reinforcement are carried out using a design acceleration (k) equal to either the full PGA (A) or an amplified PGA (k = (1.45 - A)A for A < 0.45g, k = A for A > 0.45g), so that displacement is not assumed to take place on the sliding surface. At the same time, the resistance from polymer reinforcement is taken as its short-term tensile strength, to take account of the very short-term duration of the increased applied load under the seismic design case.

4 THE UCSD SHAKING TABLE TEST

4.1 Description of the shaking table test

Sander et al (2014) describe a full-scale shaking table test carried out at the University of California San Diego (UCSD) in the spring of 2013. Figure 9 shows a cross-section through the wall, which was 6.1m high with a vertical modular block facing and compacted sand fill. The geosynthetic reinforcement consisted of a uniaxial HDPE geogrid, of length 4.27m based on achieving an L/H ratio = 0.7. The reinforcement was attached to the facing blocks using mechanical connectors. Properties of the fill and the geosynthetic reinforcement are given in Figure 9. The interior width of the container holding the model was 4.62m, resulting in a total model weight of 827t. The model was extensively instrumented to measure reinforcement strain, stresses, displacements and accelerations during the subsequent shaking.

The wall was shaken by a series of scaled real earthquake records (Kobe 1995 and Northridge 2010), with maximum input acceleration at the base of 0.55g, although the wall had been designed for a maximum PGA of 0.7g. The wall performed extremely well. A total of 41 motions were applied to the wall resulting in permanent deformations of only 56mm at the crest, reducing to 3mm at the toe. Two significant cracks developed early in the shaking and progressively widened ultimately to reach more than 30mm in width - one at the back of the reinforced soil zone and one approximately 300mm from the rear boundary - which suggests that a failure wedge developed in the retained zone of the backfill.

4.2 Back-analysis of UCSD shaking table test using the two-part wedge approach

The cross-section of the UCSD retaining wall has been back-analysed using the two-part wedge technique outlined in Section 3.3. Dimensions, reinforcement layout and soil properties have been used as given in Figure 9. The two-part wedge method used was set up based on the partial load and resistance factors given by AASHTO/LRFD, which is described in detail by Dobie (2015). Calculation results are expressed in terms of the capacity-demand ratio (CDR), defined as the ratio of factored resistance to factored disturbing force, with a target of CDR > 1.0 for all wedges checked in order to obtain a satisfactory result. With reference to Figure 8 (right), the force Z^*_i is the disturbing force for the seismic case after the component forces have been factored according to the LRFD principles. Figure 7 (left) indicates the resistances from the reinforcement ($R^*_i = T_1 + T_2 + T_3$) after application of the defined resistance factors. Based on this, CDR = R^*_i/Z^*_i for any two-part wedge for the seismic case.

Figure 10 shows the back-analysis result for the case of PGA = 0.55g with the fill shear strength = 38° (the constant volume strength). The values of Z and R may be seen in the table



Figure 9. The UCSD shaking table test section indicating the main features and dimensions.

).280 -	4.550 -	Base leve Steepest Sliding on	0.203 unreinforced we grid at this leve		Calcul 33 degrees 71≥ 1.0, OK	ate ; CDR	= 1.224≥ 1.0, OK	
	1111777.	Angle	Z	R	CDR	^		
	11111111	2.0	-108.837	10.240	-			
1111111	1111111111	5.0	-78.731	10.240	-		No. of wedges not OK:	
	· · · · · · · · · · · · · ·	8.0	-51.933	10.240	-		3	
		11.0	80.529	79.662	0.989		Load Case:	
	<u>anne //</u>	14.0	97.915	79.662	0.814		47.7.7.1.7.7.7.1.1	
		17.0	113.183	110.435	0.976		C Static	
		20.0	126.561	149.083	1.178			
monorine, 5		23.0	138.229	149.083	1.079			
the second second	1 2.4.5 -	26.0	148.330	198.719	1.340		G Dunamia	
		29.0	156.978	218.505	1.392		v• Dynamic	
0_1	Earthquake Ah = 0.550 g	: Av = 0.000g 32.0	164 259	248 266	1511	Y		

Figure 10. Back-analysis of the UCSD shaking table test using two-part wedge based on AASHTO/ LRFD.

Table 3.	Summary of the	back-analysis of the	UCSD shaking table test.
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Analysis result	PGA = 0.55g		PGA = 0.7g	
CDR (min) for $\phi'_{cv} = 38^{\circ}$	0.814	$(\theta_{cr} = 14^{\circ})$	0.450	$(heta_{cr} = 14^\circ)$
CDR (min) for $\phi'_{cv} = 42^{\circ}$	1.593	$(\theta_{cr} = 23^{\circ})$	0.722	$(heta_{cr} = 14^\circ)$

on the right-hand side of the figure, with the resulting CDR given in the next column. The lowest value is CDR = 0.814 at a wedge angle of θ = 14°. There are three wedges with CDR < 1.0, all at a low angle indicated by darker lines on the diagram on the left of Figure 10. It should be noted that CDR < 1.0 does not necessarily indicate failure, but it does indicate that the margin against failure has dropped below the required target. Similar analyses were carried out for PGA = 0.7g (the design acceleration), also for fill shear strength = 42° (peak).

The results are summarised in Table 3, which includes the angle of the base of Wedge 2 for the critical case (θ_{cr}) . The critical wedge angle for high accelerations like these is always very low, as shown on the left-hand side of Figure 10. Based on the CDR values for 0.55g, the excellent observed behaviour of the wall can be explained, together with the observation noted above that a wedge mechanism may have been developing in the backfill behind the reinforced soil zone. The diagram on the left-hand side of Figure 10 does not show Wedge 1, but it would have an upward inclination of about 45° starting from the right-hand end of the line defining Wedge 2.

4.3 Sensitivity analysis of the internal stability to the vector direction of the PGA

The UCSD wall section has been used to investigate the effect of modelling the PGA as a vector, with vertical and horizontal components which have a combined resultant equal to a specific acceleration value. This has been done for four backfill strengths ($\phi' = 30^\circ$, 34°, 38° and 42°). The procedure used was as follows, using the same wall dimensions and reinforcement layout as the UCSD test wall. For horizontal acceleration only, the acceleration value was adjusted until the minimum CDR was just > 1.0, referred to as k. Then k was applied at a series of vector angles (5°, 10°, 15°, etc.) with horizontal and vertical components which had a resultant equal to k. Therefore, the acceleration was the same in all analyses, but its direction in relation to the horizontal was gradually adjusted, looking at both the upward (-ve) and downward (+ve) vector directions. The minimum CDR was determined for each vector angle.

The results of this analysis are shown in Figure 11, which plots the minimum CDR against the vector angle (-ve vector angle is upwards). The value of k becomes higher as ϕ' increases for the wall arrangement in Figure 9 and, when k and ϕ' are high, minimum CDR drops significantly as the vector angle increases in the upward direction from zero (i.e. purely horizontal acceleration). In the case of $\phi' = 42^{\circ}$ and k = 0.636g, CDR drops as low as 0.64, almost certainly resulting in a potentially unsafe condition. This drop-off in CDR with increasing



Figure 11. Sensitivity of minimum CDR to the vector angle of the PGA.

upward vector angle becomes less marked as k and ϕ' reduce, but the trend can still be seen. At higher accelerations, the critical vector angle is at around 20° to 30° upwards, so based on this it would be advisable to use a design acceleration with components given by $k_v \approx \pm 0.5k_h$ for this specific design case. At these high accelerations and low wedge angles, k_v up is clearly critical, so that the critical vector direction is upwards This observation is based on a very limited sensitivity analysis, so the conclusion cannot be generalised, however, it does indicate that a sensible design approach would be to investigate a range of vector angles in order to find the required layout of reinforcement.

In Eurocode 8 (CEN, 2004), the Eurocode which provides advice for geotechnical design under seismic loading conditions, Section 4.1.3.3 defines the required additional design seismic force in the vertical direction (F_V) related to the horizontal force (F_H) as $F_V = \pm 0.5F_H$ if the ratio $a_{vg}/a_g > 0.6$, and $F_V = \pm 0.33F_H$ if the ratio $a_{vg}/a_g < 0.6$. The values a_{vg} and a_g are the vertical and horizontal PGA respectively. This requirement means that even if the ground motion is taken as being purely horizontal, a vertical component of acceleration should still be included for design. These values are shown in Figure 11 as vertical lines, and the implied upward vector angles coincide very closely to the minimum CDR for the cases of high acceleration. This indicates that the advice in Eurocode 8 matches very closely with the critical design condition derived from the sensitivity analysis above. It should be noted that at these low wedge angles (θ_i), it is always k_v up which is critical.

5 CONCLUSIONS

Vertical acceleration may be high or even dominant in the epicentral area of strong earthquakes. Detailed examination of strong motion data indicates that the possibility of peak vertical and peak horizontal acceleration occurring at exactly the same point in time is low. However, the component of vertical acceleration coinciding with the peak horizontal acceleration may still be significant. Therefore, it is advisable to consider combinations of horizontal and vertical acceleration in the design of reinforced soil structures.

Many design codes use the tie-back wedge method for internal stability analysis, but this does not provide a method of calculation which can take account of the disturbing forces due to vertical acceleration. This situation can be addressed by using a two-part wedge method of analysis, in which the forces generated by vertical acceleration can be included rigorously. This limiting equilibrium method uses a search routine to find the critical failure mechanism, rather than assuming a single fixed critical mechanism which is the case with the tie-back wedge design method.

The two-part wedge method was used to back-analyse the stability of a full-scale 6m high reinforced soil retaining wall subjected to a maximum of 0.55g on a shaking table, giving very

good agreement with observed behaviour. Only horizontal acceleration was applied by the shaking table, however the wall geometry and reinforcement layout have been used to investigate the application of the input peak ground acceleration as a vector using the two-part wedge method. This analysis demonstrates that, in the case of high acceleration, as the vector angle increases, internal stability reduces reaching a minimum value at around $k_v = 0.5k_h$ (upwards) beyond which it increases. This observation is consistent with the advice given in Eurocode 8.

Based on the findings of this investigation, for the seismic design of reinforced soil structures a combination of horizontal and vertical acceleration should be included in analysis. This may be defined as a fixed maximum input acceleration at various angles to the horizontal, determined by adjusting the horizontal and vertical components. The vertical component should be considered both in the upward and downward directions, although for internal stability upward is likely to be critical.

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