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Internal stability of reinforced soil structures using a two-part wedge method

Michael Dobie
Asia Pacific Regional Manager, Tensar International Limited, Indonesia

ABSTRACT: The design of reinforced soil structures is normally divided into two stages: external stability which establishes the length of the reinforcement, and internal stability which gives the layout of reinforcement required for stability. For internal stability analysis, many published design guides use the tie-back wedge method of analysis, which relies on assuming a single critical internal failure mechanism. This results in several assumptions being required to take into account actual design conditions. An alternative method is described using a simple two-part wedge, in which a large number of possible failure mechanisms are examined. This has the advantage that far fewer assumptions are required, and in addition, the analysis may take into account complex design conditions, such as low connection strength between facing and reinforcement, variable design strength along the reinforcement and earthquake loading. A calculation model is described for the specific case of connection between reinforcement and modular block facing. The result is a design which is more efficient than obtained using tie-back wedge, but at the same time is able to examine critical design situations, such as low connection strength and earthquake loading, in detail.

Keywords: reinforced soil, design, tie-back wedge, two-part wedge, modular blocks, connection strength

1 INTRODUCTION

Reinforced soil techniques are now used widely to construct retaining walls and steepened slopes, and in many countries have become the preferred method, due to the cost savings which can be made compared to conventional construction and the versatility of the resulting structures. In general reinforced soil structures are defined as retaining walls when their face angle is steeper than 70° to the horizontal normally with a concrete facing of some type, and steepened slopes when face angle is less than 70° with a vegetated finish.

As the techniques have evolved, many methods have been established for wall design (fewer for slope design) by both national and governmental agencies. For example in the US, methods are published by AASHTO and NCMA (National Concrete Masonry Association) and in UK the method is given in British Standard BS 8006-1:2010. These methods all have two main elements to the calculation. Firstly an external stability analysis is carried out, which is used to determine the overall dimensions of the reinforced soil block, namely L as shown in Figure 1.

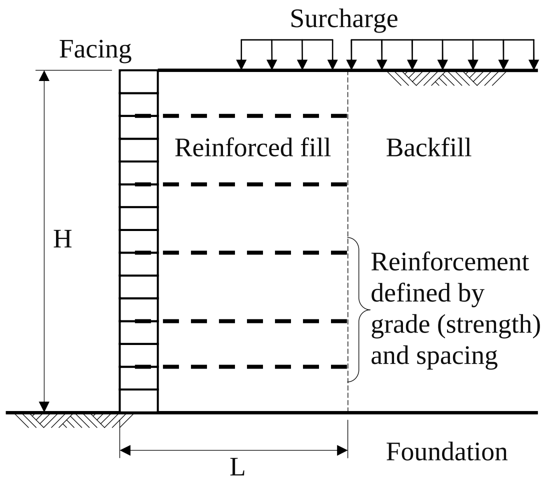

Figure 1. Reinforced soil structure main elements

The external stability check is essentially a gravity retaining wall calculation, and is much the same for all methods. The main parameter which gives differences between different methods is the choice of the wall friction angle on the back of the reinforced soil block, together with the required factors of safety. In most methods a limit is set on the ratio L/H, which will often determine the value of L, being more critical than the requirements for stability in terms of sliding, overturning and bearing resistance. External stability is not discussed further in this paper.

The second stage of the calculation is to examine internal stability, to ensure that the layout of reinforcement (grade/strength and vertical spacing) is sufficient. The internal stability calculation should also take into account design features such as the connection strength between the reinforcement and the facing. There are two main methods used to carry out the internal stability calculation: tie-back wedge and two-part wedge. The majority of published design guidelines use the tie-back wedge method (ie. AASHTO, NCMA and BS 8006-1:2010).

This paper examines the internal stability calculation procedure, firstly by outlining the tie-back wedge method together with the limitations and assumptions which are required to use this method. The remainder of the paper gives a detailed description of the two-part wedge method, including the basic principles as well as certain design conditions which may be considered of importance, as outlined in Table 1. A procedure for determining design connection strength based on connection testing is described, for the specific case of facing systems consisting of concrete modular blocks. The method assumes that geosynthetic polymer reinforcement is used.

Table 1. Conditions which may affect design

Feature	Outline
Connection	Connection strength between facing and reinforcement is generally less than the reinforcement itself
Facing temperature	High temperature at the facing may result in lower reinforcement design strength immediately behind the facing, but not further into the fill
Earthquake loading	The effect of earthquakes is modelled by additional short term loads which must be taken into account in the design

2 TIE-BACK WEDGE METHOD

2.1 *Outline of tie-back wedge method*

The tie-back wedge method is used in many design guides and codes published by both national and governmental agencies. In the descriptions and discussions which follow, specific reference is made to the AASHTO and NCMA methods, both published in USA. Both methods include procedures for carrying out seismic design, so are relevant to any location where earthquake forces must be taken into account.

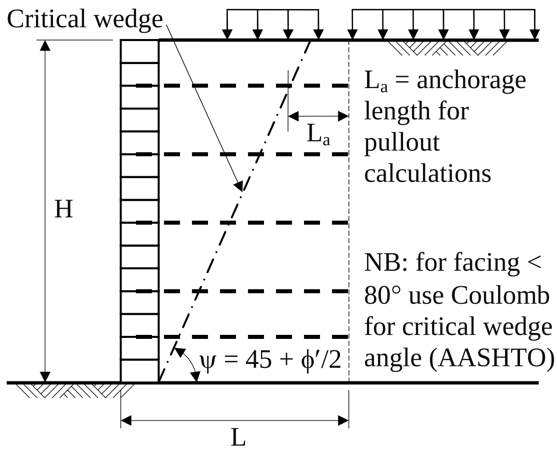

Figure 2. Defining the critical wedge (AASHTO)

The basis for the tie-back wedge method in both AASHTO and NCMA is that a single critical internal failure mechanism is assumed. In the case of AASHTO, it is the Rankine mechanism as shown on Figure 2. The Rankine wedge angle (45° + ϕ'/2) is used for any structure with a face angle between 90° and 80° to the horizontal. Then for face angles < 80° it switches to Coulomb, taking the actual facing angle in account. This results in a sudden jump in the location of the critical failure mechanism as the facing angle drops below 80°. NCMA uses Coulomb to determine the critical internal failure mechanism with $\delta = 2\phi'/3$ over the full range of facing angles, so that there is no sudden change at facing angle = 80°. It can be seen from this outline that assumptions made in two published design guidelines are already resulting in significant differences. For example for face angle = 81° and ϕ' = 34°, the angle of the critical wedge for AASHTO is 62.0° and for NCMA is 54.9°. This may lead to significant differences in the pull-out calculation (see Section 2.2).

2.2 *Tension and pull-out calculations*

The internal failure mechanism outlined in Section 2.1 is used to determine two important elements of the internal stability calculation: tensile force in the reinforcement and the available length of the reinforcement to resist pull-out.

The maximum tensile force to be resisted by the reinforcement (T_{max}) is calculated as:

$$T_{max} = K_a \times \sigma_v' \times S_v < T_{al}/FS \;\&\; T_{con}/FS \quad (1)$$

where K_a = coefficient of active earth pressure according to the mechanism defined in 2.1
σ_v' = vertical effective stress on the reinforcement including surcharges
S_v = effective vertical spacing of reinforcement
T_{al} = maximum allowable strength of the reinforcement
T_{con} = maximum connection strength
FS = required factor of safety

It should be noted that the resulting distribution of T_{max} is assumed to be the same on any vertical plane within the reinforced soil block, including directly behind the facing. T_{max} is then used to check that the tensile strength of the reinforcement and connection strength with the facing is sufficient for stability. The value of K_a is determined according to the mechanisms outlined in Section 2.1. For the same example of face angle = 81° and ϕ' = 34°, K_{ah} for AASHTO is 0.283 and for NCMA is 0.190. This difference is accentuated by choosing 81° for the facing angle, but this is not uncommon for reinforced soil structures.

The pull-out check is based on the anchorage length L_a, as shown on Figure 2. It is required that the anchorage resistance generated by L_a is greater than T_{max}. This may be stated as follows:

$$T_{max} < [2 \times L_a \times \sigma_v' \times \alpha_p \tan\phi']/FS \quad (2)$$

where α_p = pull-out interaction coefficient
ϕ' = frictional strength of the fill
σ_v' = effective stress without live load

The outline given above is for the static case. For the seismic case, the same critical wedge is assumed as shown on Figure 2, which is based on static forces only. It is possible to define a critical wedge which takes into account seismic forces, but this is not done. Therefore the anchorage length for the seismic pull-out check is based on the dimensions of the static active wedge. For the seismic tension check, it is necessary to calculate the additional force applied to the reinforcement due to seismic shaking. This additional force T_{md} is calculated in two different ways in AASHTO and NCMA, as outlined in Table 2:

Table 2. Methods of calculating T_{md}

Guide	Method
AASHTO	The mass of the active wedge shown on Figure 2 is used to calculate the total outward seismic inertia force by multiplying by the horizontal earthquake acceleration. This resulting force is distributed between the reinforcement in proportion to L_a. So T_{md} in the top reinforcement layer will be much smaller than in the lowest one.
NCMA	Additional seismic earth pressure is calculated using the Mononobe Okabe method and is distributed with 80% of the average at the top reducing linearly to 20% at the base. So T_{md} in the top reinforcement layer will be much higher than in the lowest one.

2.3 *Discussion and consequences*

The significant differences between K_{ah} and L_a in the AASHTO and NCMA methods emphasise the consequences of making assumptions to achieve a result. Although the choice of 81° for the facing angle tends to accentuate the differences, even for a vertical wall they are significant. However most reinforced soil facing systems normally are slightly inclined, so these observations are certainly valid. If it is assumed that L/H restrictions do not affect the design, then AASHTO will tend to give a denser layout of reinforcement compared to NCMA, but it will be shorter.

Likewise the different assumptions used to calculate the additional forces in the reinforcement due to earthquake loading are significant. Furthermore, it might be suggested that the critical wedge under seismic loading should be defined including the seismic forces, in which case the wedge angle will be reduced compare to Figure 2. This would tend to result in longer reinforcement length to meet the pull-out requirement.

The issues outlined in the above two paragraphs could be resolved by making the same assumptions in both design guides, however the assumptions would still be required, so that uncertainty would remain. However a far greater problem arises from the tension calculation as defined in Equation (1). As stated, this implies that the horizontal pressure distribution within the reinforced soil block is a fluid pressure, which "flows" past the reinforcement, so that the full "active" pressure is applied at the back of the facing. This is not the case, and there are plenty of cases where reinforced soil structures have had their facings removed (either be design or accident) and the fill has remained perfectly stable with little deformation. This is the case because the complete mechanism of developing failure within the reinforced soil block must take the reinforcement into account. Rankine and Coulomb are only valid for homogeneous isotropic soil masses, and the presence of the reinforcement contradicts this assumption. The method works, provided that the distribution of resistance from the reinforcement is similar to the distribution of T_{max}, ie. triangular. However this tends to result in reinforcement layouts which are very closely spaced towards the base of the structure.

A further major problem in applying Equation (1) is that when the connection strength T_{con} is significantly lower than the reinforcement allowable strength T_{al}, then T_{con} effectively must be applied over the full length of the reinforcement. This results in very inefficient use of the reinforcement. To look at it a different way, Equation (1) implies that the strength of the reinforcement buried a long way from the facing is determined by the strength of the connection at the facing, say 6m away, and this is not logical. A similar problem arises when the reinforcement has variable design strength along its length, which can be the case if higher design temperatures are considered immediately behind the facing in hot climates. In this case, the tie-back wedge method would have to apply the lower strength over the full length of the reinforcement. All of the issues outlined in this section are avoided by adopting the two-part wedge method of calculation as described in Section 3 which follows.

3 TWO-PART WEDGE METHOD

3.1 *Outline of two-part wedge method*

The basis of the two-part wedge method of analysis for internal stability is shown on Figure 3. The chosen geometry is typical of reinforced soil structures, but the method of analysis can incorporate all features shown without the need for any simplifying assumptions. As with the tie-back wedge, the method of analysis is that of limiting equilibrium, but with the important requirement that any mechanism used should be admissible (ie. can actually happen) and that all forces associated with that mechanism should be taken into account.

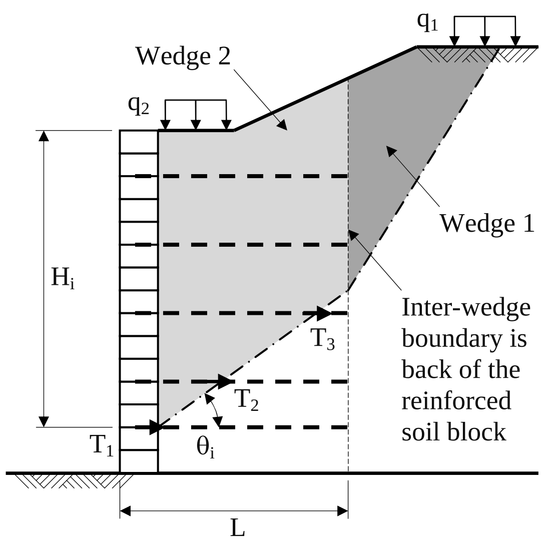

Figure 3. Basis of the two-part wedge method

It should be noted that the two-part wedge method of calculation described here was first published in a certificate granted to the author's company by a national certification body in Germany, the Deutsches Institut für Bautechnik, in Certificate No Z20.1-102 in 1995. The full certificate was restricted to various conditions suitable for use in Germany, but the method of calculation has been developed for use under much wider conditions, including various features described here. The method has been used widely and developed extensively since 1995.

The two part wedge is defined as follows:

(1) Fix a distance H_i below the top of the wall
(2) Draw a line at an angle θ_i across the reinforced soil block, defining Wedge 2
(3) Starting at the point where Wedge 2 intersects the back of the reinforced soil block, define a second wedge, Wedge 1 as shown
(4) The inter-wedge boundary is defined as the back of the reinforced soil block

The assumption that the inter-wedge boundary coincides with the back of the reinforced soil block is clearly very convenient, but might not result in the worst case. This would be a problem for lesser facing angles, but for walls where the inclination of the back of the reinforced soil block is generally > 70°, this does not introduce significant errors.

Wedge 1 is used to calculate the earth pressure forces applied to the back of the reinforced soil block, and for simple geometry and conditions, this may be replaced by the Coulomb formula (or Mononobe Okabe for the seismic design case). However for the geometry and isolated surcharge as shown on Figure 3, it is not possible to use the Coulomb formula without making some simplifying assumptions (which are given in AASHTO, for this geometry and referred to as the "broken-back" geometry). In this situation, to obtain the maximum lateral forces applied by Wedge 1 rigorously, it is necessary to use a trial wedge method in which the angle of Wedge 1 is varied until the maximum lateral thrust is obtained. This is known as the Culmann method or Coulomb sweeping wedge, and is necessary to avoid introducing simplifying assumptions to this part of the internal stability calculation.

The aim of the calculation is to make sure that the resistance provided by the facing and reinforcement which is intersected by Wedge 2 (T_1 + T_2 + T_3 as shown on Figure 3) is sufficient to avoid instability of the two wedges. Once this has been established for the wedges shown on Figure 3, another pair of wedges is selected (by adjusting H_i and θ_i) and the process is repeated. It cannot be judged in advance which pair of wedges will be critical, so it is normal to set up a search routine, which is described in Section 3.2. Details of the calculation procedure are given in Section 3.3.

3.2 *Search procedure*

In order to find the critical two-part wedge, it is necessary to search through a large number of combinations. This is normally done as shown on Figure 4. For a specific value of H_i, various values of θ_i are used so that a "fan" of wedges is checked. H_i is then adjusted and the fan of wedges repeated. Normally H_i is chosen starting at the base of the wall (H_i = H), then at each elevation where reinforcement intersects with the facing. In software developed by the author's company, θ_i is chosen at 3° intervals. In this way a large number of two-part wedges are checked.

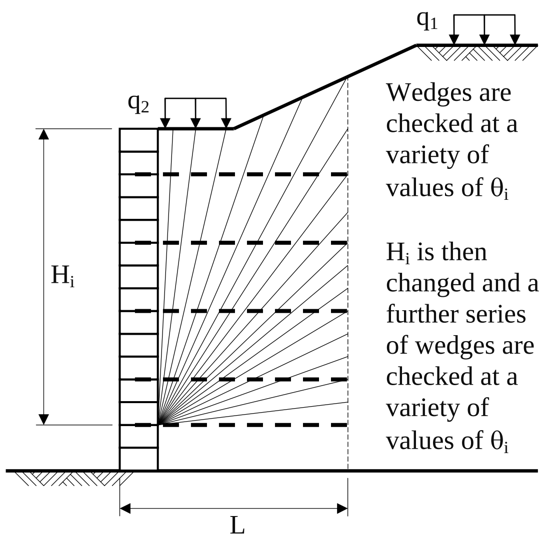

Figure 4. General search of two-part wedges

There are some special cases of two-part wedges which should be checked, as shown on Figure 5. Wedges defined by the maximum possible values of θ_i which do not intersect reinforcement may well be critical, especially if is vertical spacing is large. This check is normally carried out between all pairs of reinforcement layers, as shown on Figure 5. In the case of uniform spacing and surcharge, the critical case is the lowest wedge. However at higher levels where vertical reinforcement spacing is increased, this check may be critical again. It may also be critical if large isolated surcharges are present just behind the reinforced soil block. This check also has the benefit of ensuring that vertical spacing does not become too large.

The second check is sliding over the reinforcement, which is also required by some tie-back wedge methods (NCMA requires internal sliding checks, whereas AASHTO does not). This check may be critical in cases where the fill/reinforcement combination has a low sliding interaction factor, and is generally critical for the lowest layer of reinforcement.

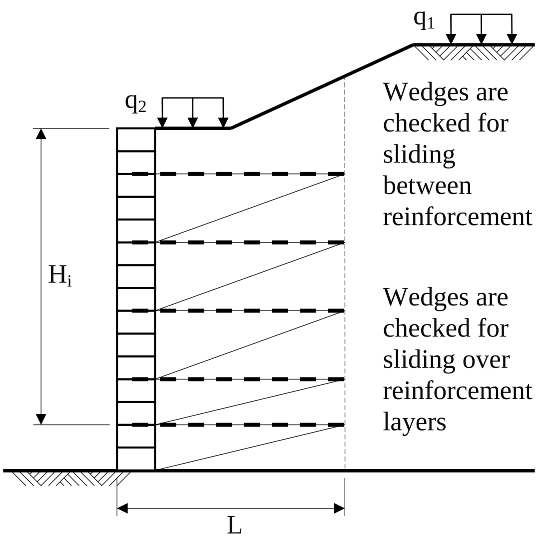

Figure 5. Special cases of the two-part wedge

3.3 *Method of calculation*

The method of calculation is shown on Figure 6. The various forces applied to Wedge 2 are calculated is outlined in Table 3.

Table 3. Forces applied to Wedge 2

Force	Comments
E_{ah}	Horizontal earth pressure force applied by retained backfill and any superimposed surcharges behind the reinforced soil block
E_{av}	Vertical component of E_{ah}
W_i	Weight of Wedge 2
Q_2	Any surcharge applied to the top of the reinforced soil block. If Q_2 is a live load, then it is not immediately obvious whether it should be included or not, so it is normal to check both with and without live load and use the critical case
R_i	Resistance on the base of Wedge 2

A simple calculation is carried out to find Z_i which is the horizontal force required to stabilise the two wedges shown. Z_i is found by resolving the forces applied to Wedge 2, as follows:

$$Z_i = \Sigma H_i - \Sigma V_i \tan(\phi' - \theta_i) \qquad (3)$$

where ΣH_i = Sum of all the horizontal forces $= E_{ah}$ in this case

ΣV_i = Sum of all the vertical forces $= W_i + Q_2 + E_{av}$ in this case

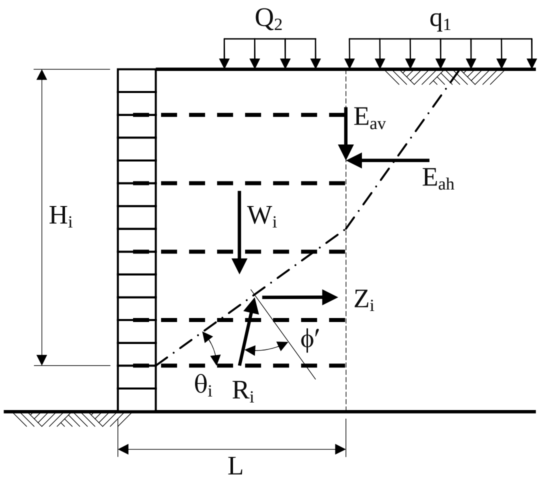

Figure 6. Calculating force required (static)

The value of Z_i found from Equation (3) is then compared to the available resistance from the reinforcement. This is shown in Figure 7, where it is assumed that two of the reinforcement layers contribute to the resistance (Layers 2 and 3). Starting with Layer 3, the pullout resistance is calculated using the same approach as Equation (2), but given as:

$$T_3 = [2 \times L_{a3} \times \sigma_v' \times \alpha_p \tan\phi']/FS \qquad (4)$$

However it is possible that the pull-out resistance might be greater than the long term design strength, given as:

$$T_3 = T_{al}/FS \qquad (5)$$

The lower value is taken as critical, then the calculation is repeated for Layer 2. The sum of T_2 and T_3 must be greater than Z_i for a satisfactory result:

$$\Sigma T_i = R_i > Z_i \tag{6}$$

The same procedure is used for all two-part wedges which intersect reinforcement.

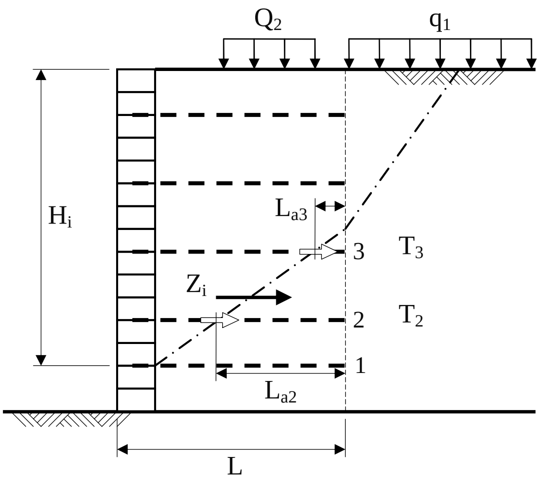

Figure 7. Calculating force available (static)

For the check of sliding on an inclined plane between reinforcement layers (as shown on Figure 5), a different approach is used, and the factor of safety is calculated as follows (but still related to the forces applied to Wedge 2 as shown in Figure 6):

$$FS = (1 - R_f \tan\theta_I)\tan\phi'/(R_f + \tan\theta_i) \tag{7}$$

where R_f = Ratio of horizontal forces to vertical forces applied to Wedge 2 = $\Sigma H_i / \Sigma V_i$

The check for sliding over reinforcement is a simple sliding check, where factor of safety is calculated as follows:

$$FS = \alpha_s \tan\phi' \, \Sigma V_i / \Sigma H_i \tag{7}$$

where α_s = sliding interaction coefficient

3.4 *Addition of seismic forces*

The procedure for seismic design is the same as for static design with regards to setting up the two-part wedge and the subsequent searches carried out. The main difference comes in the method of calculation of forces applied to Wedge 2. Additional static forces are defined to represent the inertia caused by earthquake shaking, as shown on Figure 8, with comments given in Table 4. Forces due to earthquake loading are denoted with an asterisk (*) to distinguish them from static forces. The basic approach is to assess the additional forces due to the earthquake, and add these to the underlying static forces.

Table 4. Seismic forces applied to Wedge 2

Force	Comments
E^*_{ah}	Additional horizontal earth pressure force applied by retained backfill and any superimposed surcharges due to earthquake (dynamic increment)
E^*_{av}	Vertical component of E^*_{ah}
$k_h W^*_i$	Horizontal inertia of Wedge 2* defined by a width of 0.5H from the front of the facing
$k_v W^*_i$	Vertical inertia of Wedge 2* which can act either up or down. It is not certain which will be critical, so it is normal to check both and use the critical case
Q^*_2	Surcharges applied to the top of the reinforced soil block have both horizontal and vertical inertia

Z^*_i is calculated in the same way as the static case, using Equation (3), but in this case ΣH^*_i and ΣV^*_i include the seismic load components too.

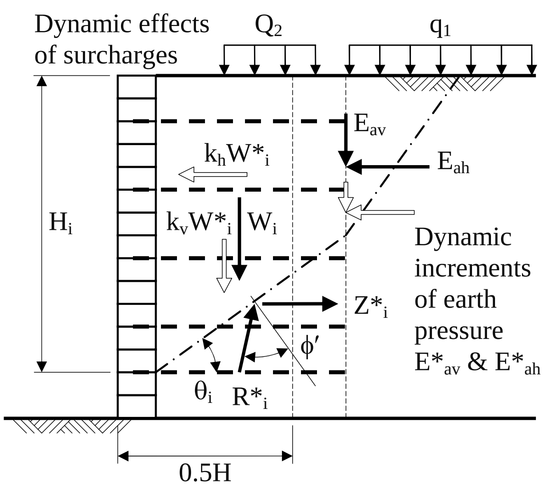

Figure 8. Calculating force required (seismic)

The calculation of the resisting force R_i is also the same as the static case, ie. following Equations (4), (5) and (6) except that the pull-out resistance is be multiplied by $(1 \pm k_v)$ and T_{al} may be taken as a short term strength appropriate to the very short term duration of loading created during earthquake shaking.

4 TWO-PART WEDGE DEVELOPMENT

4.1 *Improving the calculation model*

The two-part wedge method as described in Section 3 provides a comprehensive method of analysis of the internal stability of a reinforced soil retaining wall. Some of the assumptions required in the tie-back wedge method are avoided, such as assuming a single critical failure wedge, then basing all calculations on that single mechanism. However as described in Section 3.3, modelling of the contribution of the reinforcement to stability is still restricted to a single value of tensile strength (T_{al}) and connection strength with the facing has not been taken into account. This section describes refinements to the method of including the contribution of the reinforcement in the design, by taking advantage of the searching procedure used to find the critical design layout. In particular the concept of the "distribution of available resistance" is introduced, which provides the basis for this refinement. To help visualise what might happen when a pair of wedges fail, the mode of failure is sketched on Figure 9.

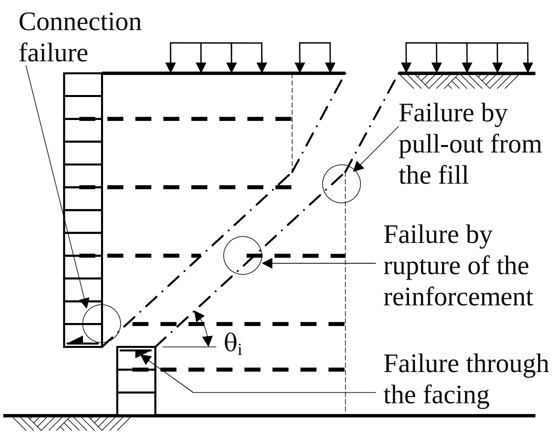

Figure 9. Likely mode of failure of two wedges

As shown on Figure 9, as the wedges slide outwards, three layers of reinforcement are involved, each with a different failure mode:

Upper — Fails due to reinforcement pulling out of the fill

Middle — Fails by rupture of the reinforcement

Lower — Fails by pulling away from the facing combined with pull-out through the fill behind the facing

In addition to the three layers of reinforcement there is also failure through the facing, in this case by sliding between two of the facing blocks, which also provides resistance. However from the point of view of the reinforcement, it is necessary to assess the available resistance at three different locations, with three different failure mechanisms. This can be done by establishing a distribution of available resistance along each layer of reinforcement as outlined in Section 4.2.

4.2 *Envelope of available resistance*

The envelope of available resistance is developed as shown in Figure 10. This is best described as a series of steps as follows below, where the vertical axis on Figure 10 is the available tensile resistance, T (in Figure 10, F as shown = $\alpha_p \tan\phi'$).

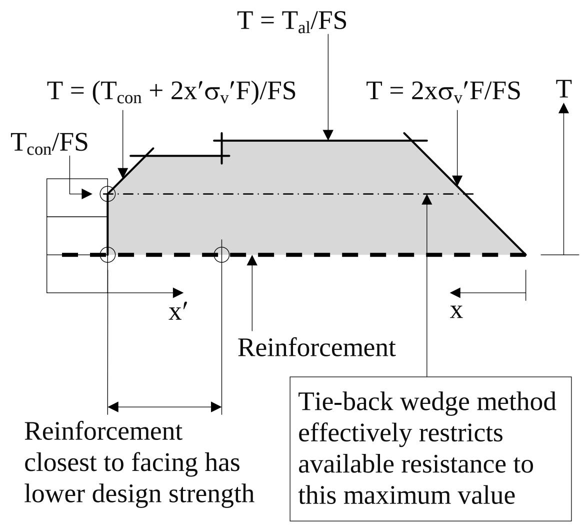

Figure 10. Envelope of available resistance

Step 1 — Starting at right end of reinforcement and moving to the left, T increases according to the pull-out equation

Step 2 — A maximum value is reached given by the tensile design strength

Step 3 — An additional design feature is shown, whereby the section of reinforcement nearest to the facing has a lower design strength, due to a higher in-soil temperature

Step 4 — The resistance at the facing is limited to the connection strength

Step 5 — Moving to the right from the facing resistance increases according to the pull-out equation

This process results in an envelope shown by the shaded area. The shape of this envelope is quite complex, however it is readily combined with the two-part wedge method of analysis as described in Section 4.3. It should be noted that Figure 10 also indicates the envelope of available resistance in the case that the tie-back wedge method is used. Effectively all of the resistance above the chain-dotted line is not used, resulting in inefficient design.

4.3 *Combining resistance envelope with two-part wedge analysis*

An envelope of available resistance may be developed for each layer of reinforcement in a structure. Figure 11 shows how these envelopes might appear. For clarity only two layers of reinforcement are shown. The sloping sections of each envelope are steeper for the lower layer of reinforcement because this slope is controlled by the vertical effective stress at the elevation of the reinforcement. This is much higher for the deeper layer.

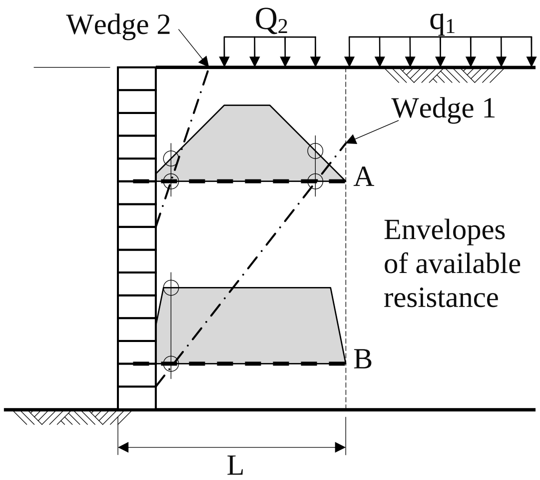

Figure 11. Analysis using available resistance

Two wedges have been added to Figure 11, and the contribution to resistance for each wedge is described as follows:

Wedge 2 Cuts Layer B near the facing, but reading up to the envelope, full tensile strength is developed. Cuts Layer A close to the buried end so that resistance comes from pull-out, and is quite low.

Wedge 1 Cuts Layer A at the same distance from the facing as Wedge 2 cutting layer B, but resistance is much smaller due to the lower connection strength and less pull-out resistance through the fill behind the facing

In the case that connection strength is relatively low near the top of the wall (as is the case with frictional connections - see Section 5), this analysis will result in fans of steep failing wedges near the top. When seismic forces are added, then failures of this type generally become more severe.

5 CONNECTION STRENGTH FOR MODULAR BLOCK WALLS

5.1 *Measuring connection strength*

Facing systems using small pre-cast concrete blocks (typically 30 to 50kg each) have become widely used over the last 10 to 15 years, and are now one of the most popular techniques for forming the facing of reinforced soil retaining walls. They are generally referred to as modular block facing systems. The blocks are stacked with mortar-less joints and the connection between the reinforcement and the facing is formed by laying the reinforcement between the blocks as they are installed. The strength of this connection is an important component of the wall design.

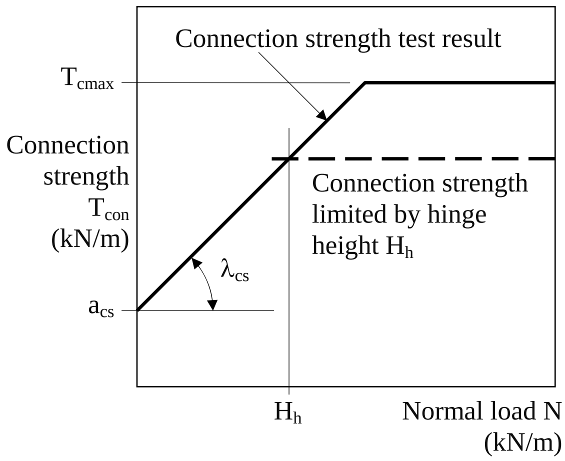

Figure 12. Results from connection testing

For any particular combination of reinforcement and modular block, it is necessary to carry out testing to measure the connection strength. The normal test standard used is ASTM D6638-07, and a typical result is shown on Figure 12, in terms of connection strength versus the normal load applied to the block at the level of the connection. There are two main types of connection technique:

Frictional	The reinforcement is clamped between the surfaces of the blocks above and below, and relies on friction generated by the normal load from the blocks above.
Mechanical	Some form of connector is incorporated at the point of connection, and the resulting strength is independent of the normal load above the point of connection

More commonly, the actual behaviour of a connection is a combination of both frictional and mechanical elements, and Figure 12 shows such a result. AASHTO recommends that purely frictional connections should not be used for walls likely to be subjected to strong seismic forces. In the explanation and discussion which follows, the following nomenclature is used:

H_h*	=	hinge height for seismic case
G_w	=	weight of blocks within hinge height
W_u	=	width of block back-to-front
D_u	=	distance from front of block to its centre of gravity
α_w	=	facing angle with respect to the vertical
T_{con}	=	connection strength
N	=	normal load at connection
a_{cs}	=	mechanical component of connection strength as measured
a_c	=	mechanical component of connection strength interpreted for static design
λ_{cs}	=	frictional component of connection strength
T_{cmax}	=	maximum connection strength
RF_{cr}	=	creep reduction factor
K_h	=	horizontal seismic coefficient
K_v	=	vertical seismic coefficient

The nature of the relationship shown on Figure 12 is similar to the Mohr Coulomb soil strength model, but with an upper limit. This may be written as given in Equation (8):

$$T_{con} = a_{cs} + N\tan\lambda_{cs} < T_{cmax} \qquad (8)$$

Without any further restriction this formula describes the solid line shown on Figure 12. For a complete interpretation it is also necessary to measure the tensile strength of the reinforcement using the same test procedure as used for the connection test, so that the results may be expressed as "efficiency". However in order to interpret this information as design strength, it is necessary to introduce further concepts, the first being "hinge height".

5.2 *Hinge height*

The hinge height is the maximum height that a stack of unsupported blocks may reach before toppling, and is used to define the maximum possible normal load (N) which may be applied at the connection level. The formula for hinge height is derived by taking moments about the front lower corner of the stack of blocks (assuming that they lean backwards, towards the fill) and is given in Equation (9). For vertical walls the hinge height is infinite (so that N would be defined by the actual height of blocks), but most modular block systems incorporate a set-back at each course so that the facing leans backwards and hinge height is finite.

$$H_h = \frac{2(W_u - D_u)}{\tan\alpha_w} \qquad (9)$$

The hinge height is equivalent to a normal load which can be plotted on Figure 12, thereby restricting the available connection strength as shown.

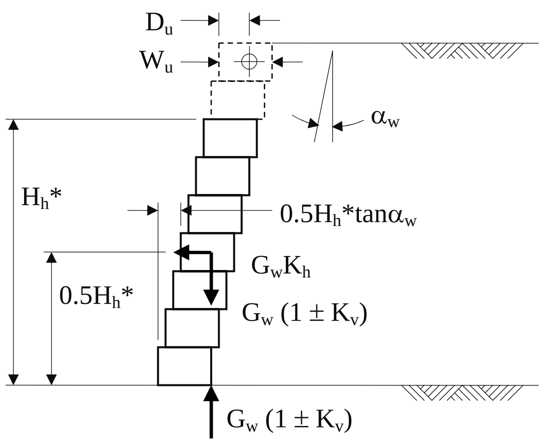

Figure 13. Defining hinge height (seismic case)

AASHTO and NCMA also use the hinge height as given in Equation (9) for the seismic design case, although it is derived for static conditions only. However it is possible to include seismic forces in the derivation of hinge height, as shown on Figure 13. Taking moments about the front lower corner of the blocks gives the expression in Equation (10) which is only appropriate for K_h acting towards the fill (ie. negative):

$$H_h{}^* = \frac{2(W_u - D_u)}{\tan\alpha_w - \dfrac{K_h}{1 \pm K_v}} \qquad (10)$$

For positive K_h, moments are taken about the back lower corner of the blocks with the assumption that tension is not permitted under the heel of the block, so that the reaction force is taken to be $2W_u/3$ from the back of the lowest block. The resulting relationship is given as Equation (11).

$$H_h{}^* = \frac{2(D_u - W_u/3)}{\dfrac{K_h}{1 \pm K_v} - \tan\alpha_w} \qquad (11)$$

These expressions are examined graphically on Figure 14, for a typical modular block with W_u = 0.3m, D_u = 0.15m and α_w = 7°.

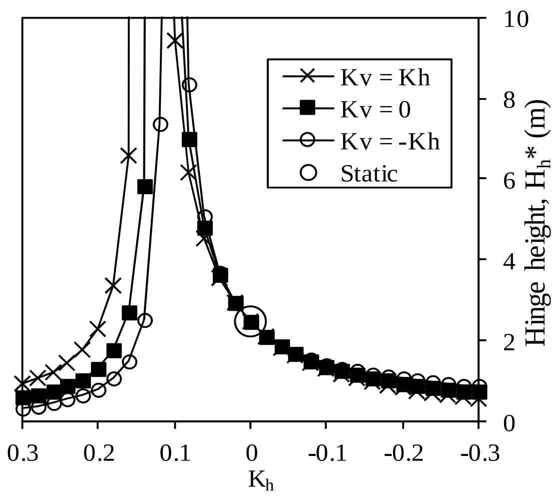

Figure 14. Hinge height versus acceleration K_h

The static hinge height for this block is 2.4m. However during an earthquake, as the accelerations cause the facing to rock backwards and forwards, the hinge height varies dramatically. It can be seen that the vertical acceleration has only a small influence on the calculated hinge height. It is clear that the hinge height under seismic conditions can become much less than the static value. For design purposes $H_h{}^*$ is calculated for both $-K_h$ and $+K_h$, and the lower value is used.

5.3 *Connection strength for design*

Test ASTM D6638-07 is a short term test, so it measures short term strength. This is suitable for seismic design, but for static design, interpretation is required to take into account the long term nature of the loading in respect to polymer reinforcement, and is based on assumptions as follows:

(a) a_{cs} is the mechanical contribution and λ_{cs} is the frictional contribution to connection strength under short term loading
(b) for static design, it is assumed that the mechanical contribution and maximum connection strength are reduced by the creep reduction factor, but the frictional contribution is unaffected. This gives: $a_c = a_{cs}/RF_{cr}$ and max $T_{con} = T_{cmax}/RF_{cr}$
(c) for seismic design the parameters measured from testing are used directly and normal load is given by $G(1 \pm K_v)$
(d) The resulting design envelopes are modified by the appropriate value of hinge height

The resulting envelopes of design connection strength for both static and seismic conditions are shown on Figure 15

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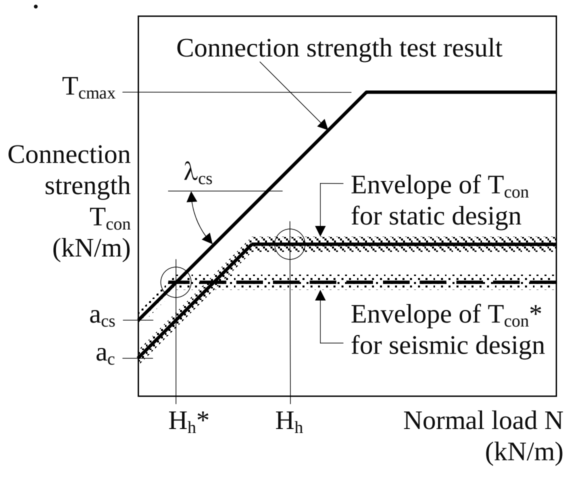

Figure 15. Connection strength for design

This procedure appears to be quite a severe interpretation compared to using static hinge height for seismic design. However because it is combined with the improved two-part wedge model as described in Section 4, then it will not dominate design, but it will penalise connections which are mainly frictional, especially under earthquake loading. However in this situation frictional connections are undesirable anyhow.

6 CONCLUSIONS

A This paper describes a two-part wedge method of analysis of the internal stability of reinforced soil structures, which is based on complete mechanisms in which all forces are taken into account, and uses a search procedure to establish the critical case.

B The contribution of reinforcement is defined in terms on an envelope of available resistance, thereby making allowance for design features such as connection strength with the facing and variable reinforcement strength.

C A method of interpreting modular block connection tests is given, in order to derive design parameters for both static and seismic loading, including the use of seismic hinge height.

D The paper only describes the method of calculation. In order to create a design method, it is necessary to define a number of parameters to be used such as: wall friction angle, factors of safety required, and seismic parameters in the case of earthquake design. This may be done using either traditional lumped safety factor methods, or limit state methods by using partial load and material factors.

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