

Design of reinforced soil structures using a two-part wedge mechanism based on AASHTO/LRFD

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ABSTRACT

Design methods for reinforced soil structures may be divided into three components: a calculation procedure, material parameters and factors. The calculation procedure normally has two stages: external stability and internal stability. A calculation procedure is outlined for internal stability analysis based on a two-part wedge mechanism, which has the benefit of requiring very few assumptions to find a solution. In particular, no assumptions are made concerning the critical failure mechanism, and instead a large number of possible two-part wedge mechanisms are searched. This approach permits a number of important design situations to be modelled in a fundamentally correct way, including connection strength between the facing and the reinforcement and earthquake loading. The method has been formulated using the partial factors and general recommendations given in the AASHTO/LRFD guideline, and the resulting designs are in general very economical. However specific weaknesses such as low connection strength between reinforcement and facing may give a critical design condition.

1. INTRODUCTION

Reinforced soil structures are defined as retaining walls when their face angle is steeper than 70° to the horizontal, normally with a concrete facing of some type. As these techniques have evolved over the last 30 years, many methods have been established for reinforced soil retaining wall design by both national and governmental agencies. For example in the US, methods are published by AASHTO and NCMA (National Concrete Masonry Association) and in UK a method is given in British Standard BS 8006-1:2010. Design of reinforced soil structures of this type is carried out in two stages. Firstly an external stability analysis is carried out, which is used to determine the overall dimensions of the reinforced soil block, namely B as shown in Figure 1. The external stability check is essentially a gravity retaining wall calculation, and is much the same in all codes and guidelines. In most methods a limit is set on the ratio B/H, which will often determine the value of B, being more critical than the requirements for stability in terms of sliding, overturning and bearing resistance. External stability is not discussed further in this paper.



Figure 1. Reinforced soil structure main elements.

The second stage of the calculation is to examine internal stability, to ensure that the layout of reinforcement (grade/strength and vertical spacing) is sufficient. The internal stability calculation should also take into account design features such as the connection strength between the reinforcement and the facing. There are two main methods used to carry out the internal stability calculation: tie-back wedge and two-part wedge. The majority of published design guidelines use the tie-back wedge method (eg. AASHTO, NCMA and BS 8006-1:2010).

This paper examines the method of calculation for internal stability, firstly by outlining the tie-back wedge method together with some of the limitations and assumptions which are required to use this method. The remainder of the paper gives a detailed description of the two-part wedge method, including the basic principles as well as certain design conditions which may be considered of importance, as outlined in Table 1. The final section outlines the partial factors defined by AASHTO/LRFD which may be combined with the two-part wedge method of calculation to create a comprehensive method for the design of reinforced soil retaining walls. The method assumes that geosynthetic polymer reinforcement is used.

Table 1.	Conditions	which	mav	affect	desian
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Feature	Outline
Connection	Connection strength between facing and reinforcement is generally less
	than the reinforcement itself
Earthquake loading	The effect of earthquakes is modelled by additional short term loads
	which must be taken into account in the design

2. TIE-BACK WEDGE METHOD

2.1 Outline of tie-back wedge method

The tie-back wedge method is used in many design guides. In the descriptions and discussions which follow, specific references are made to the AASHTO and NCMA methods, both published in USA. Both methods include procedures for carrying out seismic design, so may be relevant to any location where earthquake forces must be taken into account.



Figure 2. Defining the critical wedge (AASHTO)

The basis for the tie-back wedge method in both AASHTO and NCMA is that a single critical internal failure mechanism is assumed. In the case of AASHTO, it is the Rankine mechanism as shown on Figure 2. The Rankine wedge angle (45° + $\phi'/2$) is used for any structure with a face angle between 90° and 80° to the horizontal. Then for face angles < 80° it switches to Coulomb, taking the actual facing angle in account. This results in a sudden jump in the location of the critical failure mechanism as the facing angle drops below 80°. NCMA uses Coulomb to determine the critical internal failure mechanism with $\delta = 2\phi'/3$ over the full range of facing angles, so that there is no sudden change at facing angle = 80°. It can be seen from this outline that assumptions made in two published design guidelines are already resulting in significant differences. For example for face angle = 81° and $\phi' = 34^\circ$, the angle of the critical wedge for AASHTO is 62.0° and for NCMA is 54.9°. This may lead to significant differences in the pull-out calculation (see Section 2.2).

2.2 Tension and pull-out calculations

The internal failure mechanism outlined in Section 2.1 is used to determine two important elements of the internal stability calculation: tensile force in the reinforcement and the available length of the reinforcement to resist pull-out. The maximum tensile force to be resisted by the reinforcement (T_{max}) is calculated as:

$$T_{max} = K_{ah} \times \sigma_{v'} \times S_{v} < T_{al} \times \phi \& T_{con} \times \phi$$

[1]

 K_{ah} horizontal coefficient of active earth pressure according to the mechanism defined in 2.1

- vertical effective stress on the reinforcement including surcharges σ_{v}'
- effective vertical spacing of reinforcement Sv =
- Tal maximum allowable strength of the reinforcement =
- maximum allowable connection strength between reinforcement and facing T_{con} =
- partial material factor (as defined in AASHTO/LRFD) = φ

It should be noted that the resulting distribution of Tmax is assumed to be the same on any vertical plane within the reinforced soil block, including directly behind the facing. Tmax is then used to check that the tensile strength of the reinforcement and connection strength with the facing is sufficient for stability. The value of Kah is determined according to the mechanisms outlined in Section 2.1. For the same example of face angle = 81° and ϕ' = 34°, K_{ah} for AASHTO is 0.283 and for NCMA is 0.190. This difference is accentuated by choosing 81° for the facing angle, but this is not uncommon for these structures.

The pull-out check is based on the anchorage length La, as shown on Figure 2. It is required that the anchorage resistance generated by L_a is greater than T_{max} . This may be stated as follows:

$$T_{max} < 2 \times L_a \times \sigma_v' \times \alpha_p tan \phi' \times \phi$$

where α_p

where

- pull-out interaction coefficient = frictional strength of the fill ¢′
- vertical effective stress without live load = σ_{v}

The outline given above is for the static case. For the seismic case in AASHTO, the same critical wedge is assumed as shown on Figure 2, which is based on static forces only. Therefore the anchorage length for the seismic pull-out check is based on the dimensions of the static active wedge.

2.3 Discussion and consequences

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The significant differences between Kah and critical wedge angle values in the AASHTO and NCMA methods emphasise the consequences of making assumptions to achieve a result. Although the choice of 81° for the facing angle tends to accentuate the differences, even for a vertical wall they are significant. However most reinforced soil facing systems normally are slightly inclined, so these observations are valid. If it is assumed that B/H restrictions do not affect the design, then AASHTO will tend to give a denser layout of reinforcement compared to NCMA, but it will be shorter.

Likewise the assumptions used to calculate the additional forces in the reinforcement due to earthquake loading are significant. It might be suggested that the critical wedge under seismic loading should be defined including the seismic forces, in which case the wedge angle will be reduced compared to Figure 2. This would result in longer reinforcement length to meet the pull-out requirement in Equation 2.

The issues outlined in the above two paragraphs could be resolved by making the same assumptions in both design guides, however the assumptions would still be required, so that uncertainty would remain. However a far greater problem arises from the tension calculation as defined in Equation 1. As stated, this implies that the horizontal pressure distribution within the reinforced soil block is a fluid pressure, which "flows" past the reinforcement, so that the full "active" pressure is applied at the back of the facing. This is not the case, and there are plenty of cases where reinforced soil structures have had their facings removed (either be design or accident) and the fill has remained perfectly stable with little deformation. This is the case because the complete mechanism of developing failure within the reinforced soil block must take the reinforcement into account. Rankine and Coulomb are only valid for homogeneous isotropic soil masses, and the presence of the reinforcement contradicts this requirement. The method works, provided that the distribution of resistance from the reinforcement is similar to the distribution of T_{max}, ie. triangular. However this tends to result in reinforcement layouts which are very closely spaced towards the base of the structure.

A further major problem in applying Equation 1 is that when the connection strength T_{con} is significantly lower than the reinforcement allowable strength Tal, then T_{con} effectively must be applied over the full length of the reinforcement. This results in very inefficient use of the reinforcement. To look at it a different way, Equation 1 implies that the available resistance of the reinforcement buried a long way from the facing is determined by the strength of the connection at the facing, say 6m away, and this is not logical. All of the issues outlined in this section are avoided by adopting the two-part wedge method of calculation as described in Section 3 which follows.

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[2]

3. TWO-PART WEDGE METHOD

3.1 Outline of the two-part wedge method

The basis of the two-part wedge method of analysis for internal stability is shown on Figure 3. The geometry is typical of reinforced soil structures, but the method of analysis can incorporate all features shown (ie. berm, slope above the wall, isolated surcharge) without the need for any simplifying assumptions, which is not possible using the tie-back wedge method, which does require simplifying assumptions. The method of analysis is that of limiting equilibrium, but with the important requirement that any mechanism used should be admissible (ie. can actually happen) and that all forces associated with that mechanism should be taken into account.

The two part wedge is defined as follows: fix a distance z_i below the top of the wall, then draw a line at an angle θ_i across the reinforced soil block, defining Wedge 2. Starting at the point where Wedge 2 intersects the back of the reinforced soil block, define a second wedge, Wedge 1 as shown, with the inter-wedge boundary defined as the back of the reinforced soil block (RSB).



Figure 3. Basis of the two-part wedge method

Wedge 1 is used to calculate the earth pressure forces applied to the back of the RSB, and for simple geometry and conditions, this may be replaced by the Coulomb formula (or Mononobe Okabe for the seismic design case). However for the geometry and isolated surcharge as shown on Figure 3, it is not possible to use the Coulomb formula without making simplifying assumptions. In this situation, to obtain the maximum lateral forces applied by Wedge 1 rigorously, it is necessary to use a trial wedge method in which the angle of Wedge 1 is varied until the maximum lateral thrust is obtained (Culmann wedge method) as mentioned in AASHTO/LRFD.

The aim of the calculation is to make sure that the resistance provided by the facing and reinforcement which is intersected by Wedge 2 ($T_1 + T_2 + T_3$ as shown on Figure 3) is sufficient to ensure stability of the two wedges.

3.2 Two-part wedge search procedure

In order to find the critical two-part wedge, it is necessary to search through a large number of combinations of wedges. This is normally done as shown on Figure 4 (left). For a specific value of z_i , various values of θ_i are used so that a "fan" of wedges is checked. z_i is then adjusted and the fan of wedges repeated. Normally z_i is chosen starting at the base of the wall (where $z_i = H$, the total wall height), then at each elevation where reinforcement intersects the facing.

Special cases of two-part wedges are checked, as shown on Figure 4 (right). The first are wedges defined by the maximum possible values of θ_i which do not intersect reinforcement. This check is normally carried out between all pairs of reinforcement layers and ensures that vertical spacing does not become too large. Generally the critical case is the lowest wedge, but higher wedges may be critical if vertical reinforcement spacing is increased or large surcharges are present behind the RSB. The second check is sliding over the reinforcement, which is generally critical for the lowest layer of reinforcement especially when the fill/reinforcement combination has a low sliding interaction factor.



Figure 4. Search routines used with two-part wedge method

Compound wedges entirely inside the RSB are not checked in the method described in this paper, such wedges normally being considered in the design of reinforced slopes (facing angle < 70°).

3.3 Method of calculation

The method of calculation is shown on Figure 5. The principle of AASHTO/LRFD is that partial load and material factors are applied to the various components in the stability calculation. In the nomenclature used here, partial load factors are indicated by the term γ (eg. γ_{EV} is the partial load factor on the vertical load of the reinforced soil block). Values of the partial load and material factors used are summarised in Section 4 of this paper. The factored total force required to stabilise any two-part wedge is Z_i and the factored total resistance from the reinforcement and facing is R_i . The ratio R_i/Z_i is termed the "capacity demand ratio" or CDR and the requirement is that CDR > 1.0 in any calculation.



Figure 5. Calculating forces required and available resistance (static and seismic)

The various forces applied to Wedge 2 are shown on Figure 5 (left) and are defined in Table 2. The system of forces may be resolved to find Z_i , the factored driving force which must be resisted by the reinforcement, which is given as follows (ΣH_i & ΣV_i denote sum of all horizontal & vertical forces):

$$Z_{i} = \Sigma H_{i} - \Sigma V_{i} \tan(\phi' - \theta_{i})$$
[3]

Available resistance from the reinforcement may come from either pull-out or rupture. For layer 3:

$$\Gamma_3 = \text{smaller of:} \quad \phi \times 2 \times L_{a3} \times \sigma_{v'} \times \alpha \times C_i \times \tan \phi' \times (1 \pm K_v) \text{ (pull-out)} \quad \text{or} \quad \phi \times T_{al} \text{ (rupture)}$$
[4]

Where σ_{v} is the mean vertical effective stress along L_{a3}, C_i is the pull-out interaction factor and T_{al} is the design strength of the reinforcement. This check is carried out for each layer of reinforcement which intersects the base of Wedge 2, with the total given as:

$$\Sigma T_i = R_i > Z_i$$

Table 2. Forces applied to Wedge 2

Force	Static	Seismic (subscript "e" denotes seismic force)
γенEah	Horizontal earth pressure force applied	Additional horizontal earth pressure force applied on
	on back of reinforced soil block (RSB)	back of RSB due to earthquake (γ _{EQ} E _{aeh})
γенEav	Vertical component of YEHEah	Vertical component of γ _{EQ} E _{aeh}
γ _{EQ} k _h W _{ei}		Horizontal inertia of Wedge 2 _e defined by a width of
		0.5H from the front of the facing
γevWi	Weight of Wedge 2	Vertical inertia of Wedge 2 _e either up or down.
γllQ2	Surcharge applied to the top of the	Surcharges applied to the top of the reinforced soil
	reinforced soil block, live or dead load	block have both horizontal and vertical inertia
Pi	Resisting force on base of Wedge 2	Resisting force on base of Wedge 2

For sliding on an inclined plane between reinforcement layers (as shown on Figure 4 right), a different approach is used, and the check is carried out as follows (but still related to the forces applied to Wedge 2 as shown in Figure 5 left):

$$CDR = (1 - R_f \tan \theta_i) \tan \phi' / (R_f + \tan \theta_i)$$

Where R_f is the ratio of the factored horizontal forces to the factored vertical forces, = $\Sigma H_i / \Sigma V_i$. Sliding over reinforcement is checked as follows (C_{ds} is the sliding interaction coefficient):

$$CDR = (C_{ds} \tan \phi' \times \Sigma V_i) / \Sigma H_i$$

3.4 Improving the method of calculation

The two-part wedge method outlined above provides a comprehensive method of analysis of the internal stability of a reinforced soil retaining wall. However as described in Section 3.3, modelling of the contribution of the reinforcement to stability is still restricted to a single value of tensile strength ($\phi \times T_{al}$) and connection strength with the facing has not been taken into account. This section describes refinements to modelling the contribution of the reinforcement, by taking advantage of the searching procedure used to find the critical design layout. In particular the concept of the "distribution of available resistance" is introduced, which provides the basis for this refinement. To help visualise what might happen when a pair of wedges fail, the mode of failure is sketched on Figure 6.



Figure 6. Likely mode of failure of two wedges

[6]

[5]

[7]

As shown on Figure 6, as the wedges slide outwards, three layers of reinforcement are involved, each with a different failure mode:

- Upper Fails due to reinforcement pulling out of the fill
- Middle Fails by rupture of the reinforcement
- Lower Fails by pulling away from the facing combined with pull-out through the fill behind the facing

In addition to the three layers of reinforcement there is also failure through the facing, in this case by sliding between two of the facing blocks, which also provides resistance. However from the point of view of the reinforcement, it is necessary to assess the available resistance at three different locations, with three different failure mechanisms. This can be done by establishing a distribution of available resistance along each layer of reinforcement as outlined in Section 3.5.

3.5 Envelope of available resistance

The envelope of available resistance is developed as shown in Figure 7 (left). This is best described as a series of steps as follows below, where the vertical axis is the factored available tensile resistance, $\phi \times T$ (in Figure 7 left, F as shown = $\alpha \times C_i \times tan\phi'$):

- Step 1 Starting at right end and moving to the left, $\phi \times T$ increases according to the pull-out equation
- Step 2 A maximum value is reached given by the factored tensile design strength, $\phi \times T_{al}$
- Step 3 Now start at the left end
- Step 4 The resistance at the facing is limited to the factored connection strength
- Step 5 Moving to the right from the facing, resistance increases according to the pull-out equation

This process results in an envelope shown by the shaded area, which may be developed for each layer of reinforcement in a structure. Figure 7 (right) shows how these envelopes might appear. For clarity only two layers of reinforcement are shown. The sloping sections of each envelope are steeper for the lower layer of reinforcement because this slope is controlled by the vertical effective stress at the elevation of the reinforcement. This is much higher for the deeper layer.





Two wedges have been added to Figure 7 (right), and the contribution to resistance for each wedge is described as follows:

- Wedge 1 Cuts Layer B near the facing, but reading up to the envelope, full tensile strength is developed. Cuts Layer A close to the buried end so that resistance comes from pull-out.
- Wedge 2 Cuts Layer A at the same distance from the facing as Wedge 1 cutting layer B, but resistance is much smaller due to the lower connection strength and less pull-out resistance through the fill.

In the case that connection strength is relatively low near the top of the wall, this analysis will result in fans of steep failing wedges near the top, especially severe when seismic forces are added. A full description of the seismic aspects of this method of analysis is given by Dobie (2014) which develops the concept of seismic hinge height for the case where connection strength of a modular block facing system has a frictional component.

4. OUTLINE OF REQUIREMENTS GIVEN IN AASHTO/LRFD

AASHTO/LRFD provides design criteria and guidance for all forms of retaining structure, including reinforced soil structures, in a limit state format. The method of calculation defined for the internal stability analysis of reinforced soil retaining walls is the tie-back wedge method as outlined in Section 2. However the partial factors and other general requirements defined in AASHTO/LRFD may equally well be applied to the two-part wedge method of calculation as described in detail in Section 3. This provides an opportunity to combine the two-part wedge method and the requirements of AASHTO/LRFD to create an alternative approach for the internal stability analysis of reinforced soil retaining walls. This section summarises the factors required to establish the ultimate limit state (ULS).

The definition of soil strength of the reinforced and retained fills to be used for design is peak strength (ie. ϕ'_{peak} and c'_{peak} , although c'_{peak} is normally taken as zero). One term which has a significant effect on design is the value of wall friction (δ) used between the retained fill and the reinforced fill. The definition used is that δ should be the same as the inclination angle of the retained fill to the horizontal, with a maximum value of $2\phi'/3$. So for a horizontal backfill surface, $\delta = 0$. The required partial load factors, and partial material and resistance factors are given in Tables 3 and 4 respectively.

Factor	Comments	Strength I		Extreme Event I		Service I	
		Max	Min	Max	Min	Max	Min
урс	Dead weight of facing	1.25	0.9	1.25	0.9	1.0	1.0
γен	Horizontal and vertical loads on back of RSB	1.5	0.9	1.5	0.9	1.0	1.0
γεν	Vertical dead load of RSB	1.35	1.0	1.35	1.0	1.0	1.0
γes	Vertical dead loads above or behind RSB	1.5	0.75	1.5	0.75	1.0	1.0
γwa	Water load	1.0	1.0	1.0	1.0	1.0	1.0
γll	Live traffic load	1.75	0	0.5	0	1.0	1.0
γls	Live surcharge	1.75	0	0.5	0	1.0	1.0
γEQ	Additional loads due to earthquake	NA	NA	1.0	1.0	NA	NA

Table 3. Partial load factors

Table 4.	Partial	material	and	resistance	factors
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Factor	Comments	Strength I	Extreme Event I
φ	Soil strength and soil-to-soil sliding	1.0	1.0
φ	Factor on calculated bearing resistance	0.65	1.0
φ	Tension	0.9	1.2
φ	Pull-out	0.9	1.2
φ	Connection	0.9	1.2
φ	Sliding over reinforcement	1.0	1.0



Load combination A

Load combination B

Figure 8. Definition of Load Combination A and Load Combination B

The load factors in Table 3 have maximum and minimum values, both for "Strength I" (used for static design) and "Extreme Event I" (used for the seismic design case). The maximum and minimum load factors should be combined in such a way as to give the worst case for the specific design calculation being carried out. In the case of internal stability using the two-part wedge method of calculation, there are two main options as shown on Figure 8. The terms used here are "Load Combination A" (LCA) and "Load Combination B" (LCB), which have the same meaning as defined in BS 8006-1:2010. LCA fixes all load factors to their maximum values. LCB fixes the vertical loads to their minimum values and horizontal loads to their maximum values. In the two-part wedge method, LCA is critical for steep wedges and LCB for low angle wedges. It is not certain which load combination will be critical, so both are checked and the lower CDR taken as critical.

There is a special issue to be noted with regards to LCB, when the earth pressure force has a vertical component. If that component is downwards, which is the normal case as shown on Figure 8, then there is one approach which states that this component should have the same load factor as the horizontal component (shown with * on Figure 8, right), sometimes referred to as the "single source" principle. However to give the "worst case", this load factor should be set to "minimum", therefore not the same as the horizontal component. Generally codes of practice do not give specific advice on this issue, so it is up to the designer to decide whether "single source" or "worst case" is appropriate to any specific design.

5. DESIGN CASE STUDY

In order to examine and compare typical designs based on tie-back wedge and two-part wedge, a simple case of an 8m high wall has been set up as shown on Figure 9. The facing is modular block, with all design parameters and dimensions as given. The design brief requires that the design should only use two grades of reinforcement. In order to compare designs, a "cost index" (CI) is defined based on the product of the total area of each grade of reinforcement required and its relative unit manufacturing cost. The tie-back wedge method gives CI = 41.7 units, whereas the two-part wedge gives CI = 29.5 units, a saving of 29%. Figure 9 also shows the critical mechanisms for each method of calculation. The lowest CDR for the two-part wedge method is 1.038, for a wedge at 50° which exits from the back of the reinforced soil block near the top. It is significantly different to the Rankine wedge used by the tie-back wedge method.

A variant of this design is shown on Figure 10, which is the same as Figure 9, but with a 1m wide surcharge of 120 kPa located a short way behind the reinforced zone. The two-part wedge design is affected by the 120 kPa surcharge, and Cl increases to 38.2 units, whereas due to the formulation of the tie-back wedge, the design layout of reinforcement is the same as for Figure 9. The critical two-part wedge from the 0.2m level is governed by the high surcharge. At the 2m level there are two minimum CDR values, and above this the critical wedge in no longer influenced by the high surcharge.



Figure 9. Design example of 8m high modular block wall with uniform surcharge of 12 kPa



Figure 10. Design example of 8m high modular block wall with isolated surcharge of 120 kPa behind the reinforced zone

6. DISCUSSION AND CONCLUSION

A method of calculation is described for checking the internal stability of reinforced soil structures, based on a two-part wedge mechanism, in which a large number of possible failure mechanisms are searched. The method allows connection strength and earthquake loads to be taken into account. The technique has been combined with the requirements of AASHTO/LRFD to create a complete design method, as an alternative to the frequently published tie-back wedge method. The example in Section 5 indicates a saving in reinforcement of about 29%, however it should be noted that the connection efficiency for the system used in the example is 82%, which accounts for some of this cost difference.

There is a general observation that many current design methods for reinforced soil structures are over conservative based on observed performance. The method described in this paper has the potential to reduced reinforcement cost significantly, while at the same time ensuring that potential weaknesses such as low connection strength between facing and reinforcement are not critical. However if developments of this type are able to justify significant reduction in reinforcement based on the ultimate limit state (ULS), then it becomes very important that checks are also carried out for the serviceability limit state (SLS). The two-part wedge method described in this paper may the set up to provide such a check, based on the approach and recommendations given in BS 8006-1:2010.

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