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# Reinforced soil design using a two-part wedge mechanism: justification and evidence

## Conception de sol renforcé par un mécanisme de deux blocs: la justification et la preuve

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ABSTRACT Design methods for reinforced soil structures are normally divided into: external stability (defines structure dimensions) and internal stability (determines reinforcement layout). This paper examines a method of calculation which has been developed for the internal stability check based on a simple two-part wedge mechanism. The wedges are defined by a first plane across the width of the reinforced soil zone, and a second plane upwards through the retained backfill. Reinforcement intersected by the first wedge contributes to the equilibrium of forces. A large family of two-part wedges is defined, and sufficient reinforcement must be provided to ensure that all can achieve equilibrium without overloading the reinforcement. Extensive experience of using this technique indicates that the critical two-part wedge in an efficiently designed structure will normally be defined by a line crossing the reinforced soil zone at about 45 degrees, then extending through the backfill at the Coulomb angle. If seismic inertia forces are added, then the angles of both wedges will become less steep. The two-part wedge mechanism is compared with more comprehensive stability analyses, as well as observed behaviour in shaking table tests on small-scale reinforced soil walls.

RÉSUMÉ Les méthodes de conception pour les structures en sol renforcé sont normalement divisées en: stabilité externe (définit les dimensions de la structure) et de la stabilité interne (détermine l'arrangement de renforcement). Cet article examine une méthode de calcul qui a été développée pour le contrôle de la stabilité interne basée sur un mécanisme simple de deux blocs. Les blocs sont définis par un premier plan à travers la largeur de la zone de sol renforcé, et un second plan vers le haut à travers le remblai retenu. Les renforcements coupés par le premier plan contribuent à l'équilibre des forces. Une grande famille de mécanismes est définie, et un renforcement suffisant doit être prévu pour que tous les mécanismes puissent atteindre l'équilibre sans surcharger le renforcement. La grande expérience de l'utilisation de cette technique indique que le mécanisme critique dans une structure conçue de manière efficace sera normalement définie par une ligne traversant la zone de sol renforcé à environ 45 degrés, puis s'étendant à travers le remblai à l'angle de Coulomb. Si les forces d'inertie sismiques sont ajoutées, les angles des deux plans seront moins raides. Le mécanisme de deux blocs est comparé aux analyses de la stabilité plus globale, et aux comportements observés dans les essais sur murs renforcés sur une table vibrante à petite échelle.

## 1 INTRODUCTION

Design methods for reinforced soil structures are normally divided into two stages: external stability which defines the overall dimensions of the structure and internal stability which determines the layout of the reinforcement (i.e. grade and vertical spacing). This paper examines the method of calculation used for internal stability. In most published design methods for geosynthetic reinforcement, this is carried out using a method called tie-back wedge, which assumes a single critical failure mechanism, normally defined either by Rankine or Coulomb, as shown on Figure 1 (left). Due to this simple approach, many assumptions and simplifications are required in order to carry out the calculation, some of which may lead to uncertainty and over conservatism. These issues are discussed in detail by Dobie (2015).

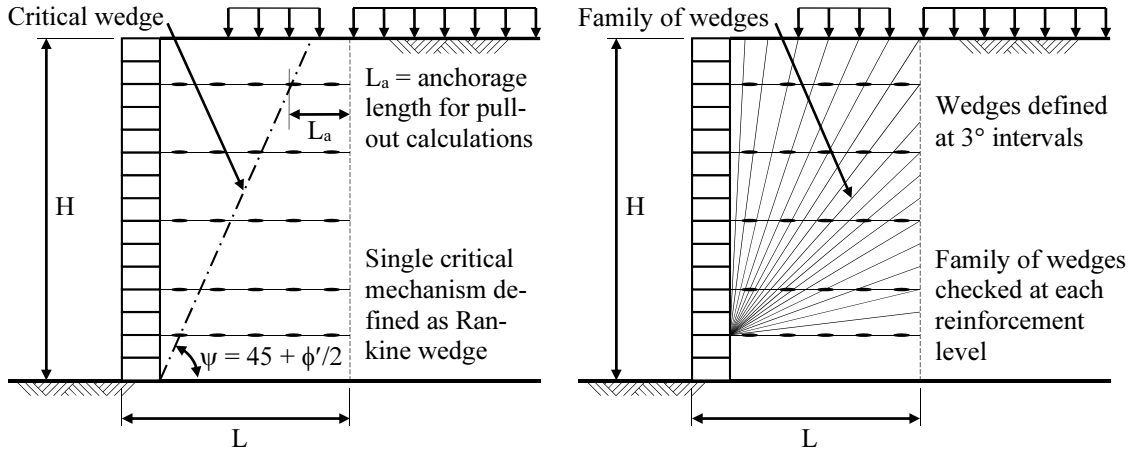


**Figure 1.** Mechanisms used to check internal stability: tie-back wedge (left) and two-part wedge (right).

Whilst the assumption of a single critical mechanism is satisfactory for a uniform homogenous soil mass, once reinforcement is included the new critical mechanism may well lie partly behind the reinforcement, and its location cannot be predicted without a method of analysis which searches for the worst case. This is called the two-part wedge method, as depicted on Figure 1 (right), which shows families of failure planes crossing the reinforced soil zone. The continuation of each mechanism would be a plane through the retained fill at an angle close to the Rankine angle. The purpose of this paper is to examine the likely and actual failure mechanisms for reinforced soil retaining walls and provide justification and evidence as to why this approach is far more realistic than tie-back wedge.

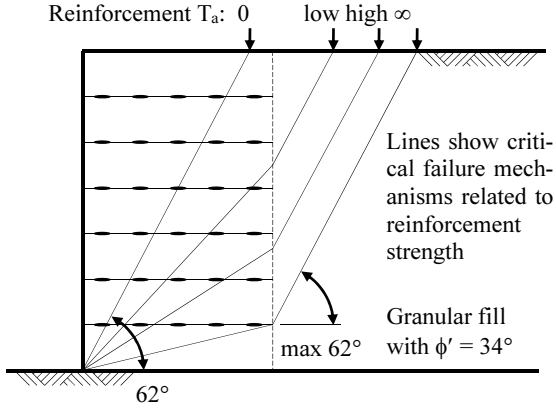


**Figure 2**. Default failure mechanisms

## 2 EXAMINING EXTREME CASES

For a simple reinforced soil retaining wall there are two extreme conditions where the critical mechanism is known in advance of calculation, as indicated on Figure 2.

Figure 2 shows a hypothetical case of a vertical retaining wall with uniformly spaced reinforcement all of the same strength, and a typical L/H ratio. The soil has strength given by $\phi' = 34°$. The first extreme, such that the critical mechanism is known by inspection, is the case where the reinforcement strength ($T_a$) is zero. In this case the critical failure mechanism is given by Rankine, and is a single wedge with base angle = $45 + \phi'/2 = 62°$. The second extreme is the case when the reinforcement has infinite strength (and pull-out capacity), so that the critical mechanism consists of a two-part wedge crossing the reinforced soil zone at an angle such that it is just bounded by the first layer of reinforcement, then continuing through the retained fill at the Rankine or Coulomb angle (depending on the angle of wall friction assumed).

For any finite value of reinforcement strength between these two extremes, the critical mechanism must also fall between these two extreme mechanisms. It can further be seen that as reinforcement strength becomes lower, the angle of the wedge crossing the reinforced zone becomes higher. Therefore in the case of very low reinforcement strength, the critical mechanism may well be a single wedge entirely within the reinforced soil zone.

There is one more case which can be determined by inspection, and this is when the reinforcement is of infinite length, but finite strength. Because the contribution to stability from the reinforcement is the same for all wedge angles, then a single wedge at the Rankine angle must again represent the critical mechanism.

A practical example of the single wedge being critical may be seen in the trial reinforced soil retaining walls reported by Bathurst et al (2001). A series of walls 3.6m high were built then surcharged until a critical condition was reached. In the case of the wall with lowest strength reinforcement (6 layers with long term strength of $T_a$ = 1.95 kN/m only), a single critical wedge was identified at about 63° to the horizontal. For the compacted sand fill used, $\phi'$ is reported as 44°, which, combined with the facing angle of 8° from vertical, gives a critical (unreinforced) wedge angle also of about 63°. In fact in this situation, with such a high $\phi'$ and relative low angle facing, the value of $K_{ah}$ (horizontal component of the active earth pressure coefficient) is only about 0.12, which is very low indeed, such that the demand for reinforcement is also low. The L/H ratio for the trial walls was 0.7, so it is almost inevitable that the observed critical mechanism was a single wedge at an angle given by Coulomb.

From the discussion above it is clear that the critical failure mechanism for a given reinforced soil retaining wall cannot be decided in advance, and can only be established by searching a large number of possible mechanisms. It would appear that the two-part wedge approach, as shown in Figure 1 (right) offers good potential. This potential is examined in the following sections, firstly based on stability analysis, and then secondly by examining results from shaking table tests.

## 3 EVIDENCE FROM STABILITY ANALYSIS

Stability analysis provides an opportunity to examine likely failure mechanisms for reinforced soil retaining walls. This could be done using slip circles, in which case it is relatively easy to set up a search routine such that a large number of possible failure surfaces are examined in order to find the surface giving the lowest factor of safety. Such search routines are common and work very well in many situations, but of course the only possible mechanism shape is a circle, and based on the preceding discussion, this may well not be appropriate for reinforced soil retaining walls.

In order to make the search more general, it is necessary to use a search based on non-circular surfaces. Figure 3 shows the section of a typical reinforced soil retaining wall with complex geometry and surcharges. By preference a search technique would be able to start with a random surface as shown, in this case formed using 13 short straight line segments. The search would then adjust the arrangement of the segments, until a surface was found giving the lowest factor of safety. With a large number of segments, this means that pretty-well any shape could be found, for example a circle, or a spiral or a straight line as depicted on Figure 3.

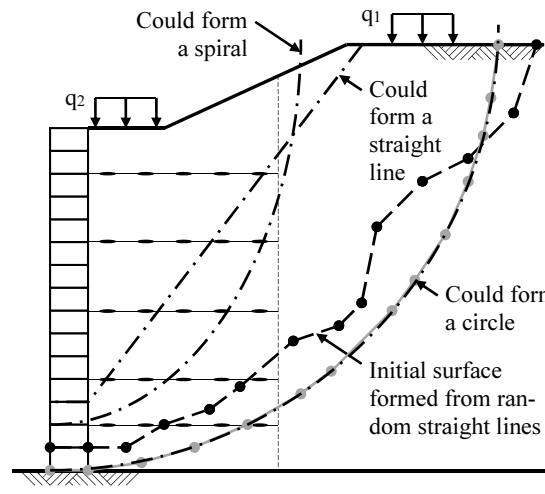


**Figure 3**. Possible non-circular failure mechanisms

The Simple Genetic Algorithm (SGA) allows such a search to be carried out with a range of slope stability analysis methods, both circular (McCombie and Wilkinson, 2002) and non-circular (Zolfaghari et al, 2005, McCombie, 2009). For this investigation, the search uses Janbu's method (Janbu, 1957). A population of potential mechanisms is randomly generated using a framework designed to give only feasible mechanisms. This population is then evolved, using processes which mimic natural selection.

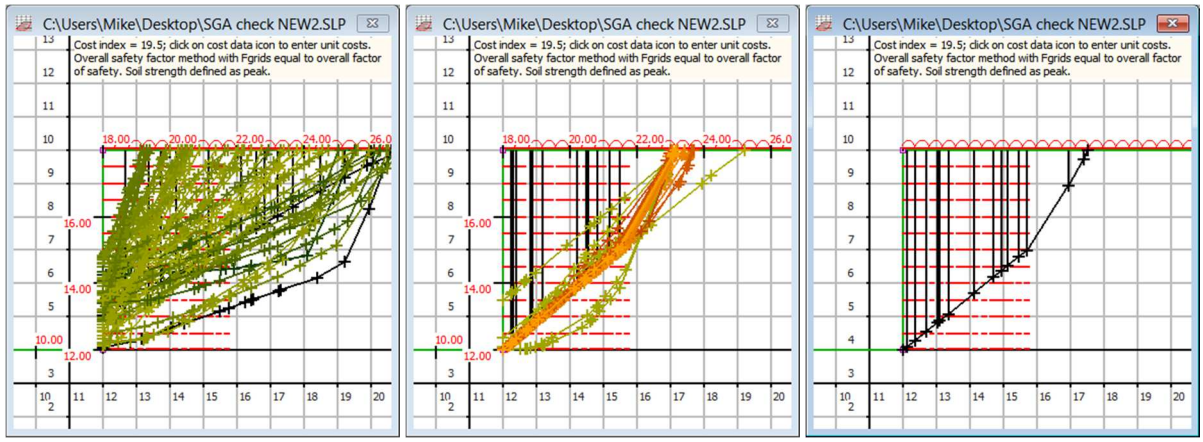


**Figure 4.** Simplified genetic algorithm process: populate (left), analyse and hunt (middle) and find critical non-circular surface (right).

Each step in the evolutionary process gives a new generation of mechanisms which becomes progressively better in terms of the chosen definition of fitness, the lowest factor of safety in this case. Because the method works with a population rather than a single mechanism, it can search for critical mechanisms in several places at once, and is ideally suited to the problem described here, in which one cannot know in advance which of the types of mechanism shown in Figures 2 and 3 will turn out to be critical.

Figure 4 shows the results of applying the Simple Genetic Algorithm to a typical vertical reinforced soil retaining wall, 6m high and with a steel mesh facing so that the facing has negligible influence on the resulting design. This is an important factor - concrete blockwork facings provide a substantial part of the retaining function in themselves, especially for low walls, and there is a danger that experimental results become almost completely useless as assessments of the reinforced soil. The fill is sand with $\phi' = 34°$. The wall was initially designed using the two-part wedge method, with partial factors as per AASHTO/LRFD (background given by Dobie, 2015) and the resulting design is very efficient, using two grades of reinforcement at a constant vertical spacing of 0.5m. The geometry was exported to the stability program, and the SGA was set up with a wide range of entry points and angles, and exit points and angles, using 15 line segments. Figure 4 (left) shows the initial population of random surfaces set up by the SGA. The middle image shows how the population of surfaces has become concentrated in the region giving the lowest factor of safety, and the right-hand image shows the critical surface, which is a perfect two-part wedge, despite the fact that it consists of 15 short segments.

A further investigation was then carried out to show the effects of seismic conditions; this shows that the two-part wedge mechanism is again found to be critical, but with the bases of the wedge sloping further back than in the static case, as shown on Figure 5.

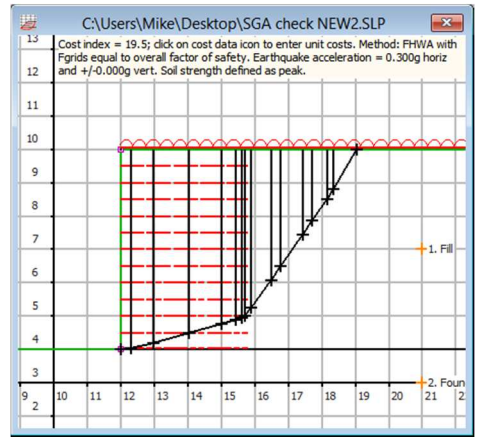


**Figure 5**. Critical mechanism under seismic loading

## 4 EVIDENCE FROM SHAKING TABLE TESTS

In order to generate loading conditions approaching failure in reinforced soil structures and therefore suit-

able to investigate the failure mechanism, one approach is to apply a high surcharge under static conditions, as was done for the test walls reported by Bathurst et al (2001). An alternative approach is to apply seismic loading using a shaking table.

Bowman et al (2011) report the results of shaking table tests carried out on one of a series of 1:5 model-scale reinforced soil retaining walls at the University of Canterbury in New Zealand. The model represents a 4.5m high wall, with model dimensions as shown on Figure 6, and with a width of 800mm. A sand fill was used with $\phi'_{cv}$ = 31° and the facing represents a full height rigid panel. The total model, with a weight of approximately 3 tonnes, was shaken in a series of stages of increasing acceleration. The excitation consisted of a sinusoidal motion in the horizontal plane at 5 Hz for 50 cycles per stage. The acceleration used for each stage increased in steps of 0.1g, until displacement at the top of the facing exceeded 100mm.

During the testing, a special high speed camera was used to record images of the area indicated on Figure 6, at the mid-height of the back of the reinforced zone. The images were analysed using a technique called particle image velocimetry (PIV) which permits the tracking of displacement fields, and the development of shear bands. The images shown in the lower part of Figure 6 show the accumulated shear strain, with the full scale bar on the right representing 40%. It can be seen from these images that the location of the shear bands is controlled by the ends of the reinforcement layers, although at the lower acceleration levels, these bands are not part of a complete failure mechanism. Failure occurred at 0.7g, and the mechanism consisted of a shear plane extending across the width of the reinforced soil zone below the lowest layer of reinforcement, then up through the retained fill at an angle of 35 to 41° to the horizontal.

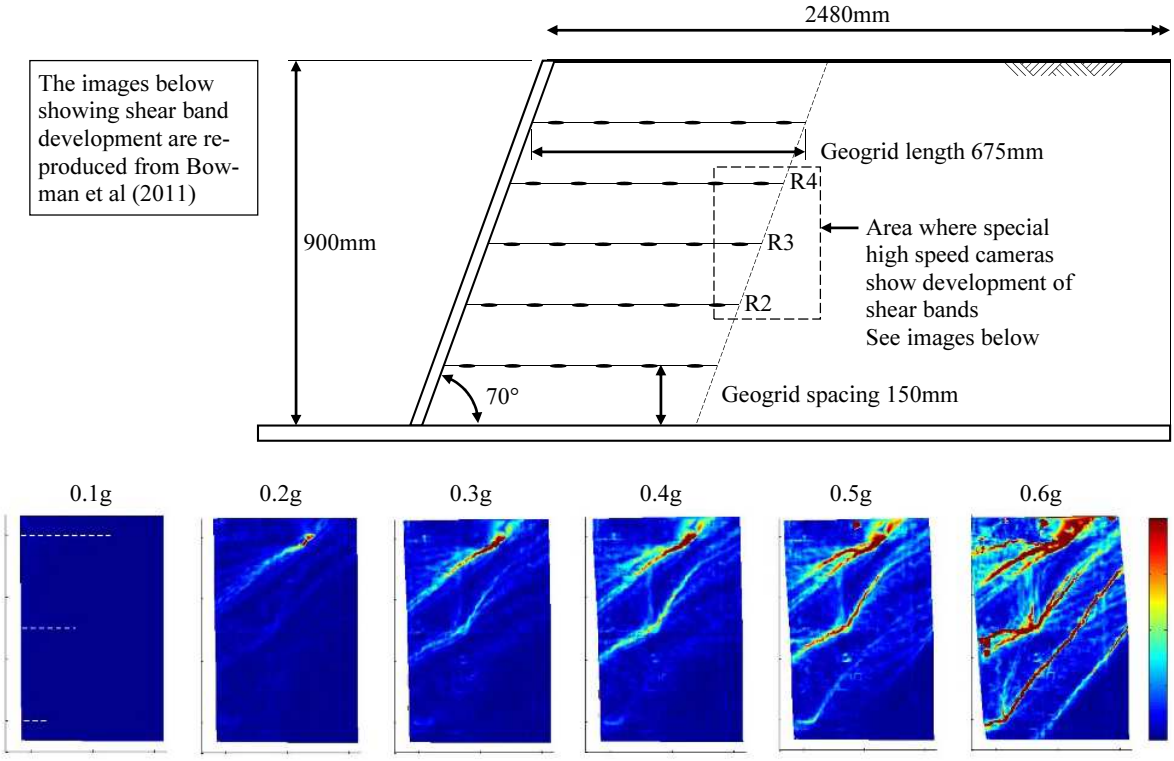


**Figure 6**. Development of shear bands in a model-scale reinforced soil retaining wall subject to seismic shaking up to 0.7g

## 5 DISCUSSION AND CONCLUSIONS

In calculating the internal stability of reinforced soil retaining walls, a large family of two-part wedges is defined, and sufficient reinforcement must be provided to ensure all can achieve equilibrium without overloading the reinforcement.

Examination of extreme cases indicates that the location of the critical two-part wedge may vary widely, depending on the strength of the reinforcement relative to the fill. In a situation where all other features and loadings are fixed, as the fill becomes stronger (ie. $\phi'$ becomes higher), the demand for reinforcement reduces and the angle of the wedge which crosses the reinforced soil zone becomes steeper. In the case of very high strength fill, the critical two-part wedge may well reduce to a single wedge entirely within the reinforced soil zone, but such a situation is generally considered to be unlikely, unless the reinforcement is relatively long for some unrelated reason.

Under normal design conditions, extensive experience of using this technique indicates that the critical two-part wedge in an efficiently designed structure will usually be defined by a line crossing the reinforced soil zone at about 45 degrees, then extending through the backfill at the Coulomb angle. If seismic inertia forces are added, then the angles of both wedges will become less steep. The two-part wedge mechanism is compared with more comprehensive stability analyses, which result in the same shape of critical failure surface. Shaking table tests on model-scale reinforced soil retaining walls also provide evidence that the critical failure mechanism is very close to being a two-part wedge, controlled by the location of the reinforcement.

The two-part wedge approach is straightforward to apply, requiring no empirically derived factors to achieve a correspondence with observed experiments or more complex methods of analysis. This transparency and accuracy means that it can be used with confidence in designs which do not replicate instrumented experimental structures; in contrast, the more empirical factors are used in a design approach, the less confidence a designer can have in extrapolating beyond established practice. The two-part wedge method has allowed very large structures to be designed and built around the world, which have performed well both in normal use and in extreme seismic conditions. This success has been critically dependent upon the transparency of the method. The comparisons examined here have shown that this success is due in no small part to the fact that the mechanisms being considered represent what actually occurs in real reinforced soil walls.

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